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Social Behaviors, Enforcement, and Tax Compliance Dynamics

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ABSTRACT: We analyze the effect of social norms and enforcement on the dynamics of taxpayer compliance. Specifically, we develop two models to evaluate the movement between classes of compliant and noncompliant taxpayers. Our analysis suggests that the effect on compliance of changing enforcement levels depends on whether the taxpayer population is initially compliant or noncompliant. Compliant populations are insensitive to changes in enforcement policies until enforcement becomes sufficiently lax, when we observe a sudden shift to high levels of noncompliance in equilibrium. In contrast, relatively noncompliant populations respond to increased enforcement by gradually increasing compliance. Then, when enforcement becomes sufficiently harsh, we find a sudden shift in equilibrium to very high levels of compliance. After the taxpayer population shifts from compliance to noncompliance, or vice versa, our models predict that returning to the previous enforcement policy will not cause the population to return to its previous state. On the whole, our models' results help explain why taxpayer compliance varies across time and across geographic regions, even under similar enforcement regimes.

Keywords: *regulation; tax compliance; agent-based modeling; nonlinear dynamics.*

Data Availability: *Contact the authors*.

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Morals cannot be legislated but behavior can be regulated.

—Martin Luther King, Jr.

I. INTRODUCTION

Enforcement and the behavior of others affect taxpayer compliance. IRS Commissioner Charles Rossotti recently stated that "the slide in audits has raised concern that honesty could suffer as fears of policing decline. If taxpayers begin to believe that others are cheating, the temptations to shave their own tax burdens may become irresistible" (Weisman 2001, A1). Rossotti's comment recognizes that tax enforcement affects social behaviors, and that these behaviors can, in turn, affect taxpayers' compliance decisions. Prior tax research concludes that individuals who know evading taxpayers are less likely to comply themselves (Vogel 1974; Spicer and Lundstedt 1976; Scott and Grasmick 1981; Grasmick and Scott 1982).[1] Likewise, the probability that a taxpayer will evade increases when the taxpayer suspects that his acquaintances are evading (Westat, Inc. 1980; Geeroms and Wilmots 1985). At the societal level, survey evidence suggests that differences in compliance across countries and geographic regions with similar or nearly identical tax and enforcement regimes stem from differences in social norms.[2] In an experiment, Alm et al. (1995) show that compliance differs across subjects from different countries. Although survey, archival, and experimental research support the contention that social norms and acquaintances' taxpaying behavior affect tax compliance, no theoretical foundation that exists suggests the form that these effects might take (Alm 1991).

We address this void in the literature by developing two models in order to provide a theoretical analysis of the effects of others' behavior, enforcement, and social norms on the dynamics of taxpayer compliance. The first, an aggregate-level model, uses a system of nonlinear differential equations to characterize the flows among three classes of taxpayers who differ in their propensities to evade: honest taxpayers, susceptible taxpayers, and evaders.[3] The flows represent the movements of taxpayers among the classes resulting from the aggregate outcomes of individual taxpayers' decisions. As the number of tax evaders increases, honest people become susceptible to evasion (honest taxpayers become increasingly likely to know evaders and to be influenced by their behavior). Similarly, the greater the proportion of honest taxpayers in the population, the greater the pressure on evaders to become honest. The level of enforcement affects the movement of taxpayers both from the susceptible class to the evading class, and from the evading class to the honest class.

To maintain tractability, we make a number of simplifying assumptions regarding aggregate behavior and we do not explicitly account for the behavior of individual taxpayers.

[1] Hite (1988) is an exception in this line of research. In a role-playing experiment using prospective jurors, she finds that peer influence has no significant effect on tax compliance.

[2] We obtained survey evidence regarding taxpayer behavior in Spain, Sweden, and the United Kingdom from de Juan et al. (1994), Vogel (1974), and Lewis (1979), respectively. For evidence regarding the United States, see Westat, Inc. (1980) and Yankelovich, Skelly, and White, Inc. (1984). These studies' results suggest that cross-cultural differences in the social acceptability of evasion may partly account for differences in observed levels of compliance. In addition, these studies found that a number of cultural variables that mediate the effect of norms on behavior (e.g., social cohesion, alienation, etc.) were significantly related to compliance behavior. Evidence also suggests that regional differences in compliance within countries may be attributable to local differences in social norms (e.g., Witte and Woodbury [1985] note an apparent effect within the United States).

[3] Mathematical biologists commonly use our approach to model epidemics (see Kermack and McKendrick 1927; Murray 1989, 611–618). Our model differs considerably from those in epidemiology, however, in that we carefully constructed the flows in our model to represent the social psychology of tax evasion. Researchers have used this class of models to investigate a number of topics in the social sciences, ranging from a model of social revolution to the spread of illegal drug use (Epstein 1997, 69–93).

To address these limitations, we create a second, complementary, model that characterizes individual taxpayers' compliance decisions. This micro-level model uses a relatively new method, agent-based modeling, which relaxes unrealistic assumptions of agent rationality and agent homogeneity, avoids the use of ''representative individuals,''[4] and facilitates the theoretical representation of agent interaction and dynamic behavior.[5] The model does not characterize aggregate behavior explicitly. Instead, the aggregate behavior emerges endogenously and dynamically from the interactions among heterogeneous agents.

In our agent-based model, we create a society of taxpayers who initially possess limited knowledge about the level of enforcement and heterogeneous reporting rules that depend on perceived enforcement severity, social norms, and acquaintances' tax compliance behavior. Our taxpayers develop and periodically update their beliefs regarding social norms and enforcement severity. Individual taxpayer behavior evolves over time, partly in response to other taxpayers' behavior.

The results of our two models contribute to an understanding of how social forces and enforcement affect compliance. Our aggregate model suggests that taxpayers tend toward one or two equilibrium states over time. The possibility of multiple equilibria is consistent with the observation that tax compliance varies over time and across geographic regions, even under similar enforcement policies. Perhaps the most important prediction is that small changes in enforcement can lead to sudden and large changes in equilibrium states from mostly compliant to mostly evading or vice versa. For example, we predict that reducing enforcement in a compliant population will lead to a gradual increase in evasion, but at some point, a sudden proliferation of tax evasion will occur, effectively leading to an evasion epidemic. Our analysis further suggests that simply returning to the previous level of enforcement will not be sufficient to stop the epidemic. Our agent-based analysis supports the aggregate model's predictions.

We also make a methodological contribution to the accounting literature by introducing two complementary modeling approaches. In contrast to standard neoclassical economics, our aggregate model builds on a social-psychological foundation and explicitly accounts for system dynamics. Our agent-based model also permits an investigation of system dynamics and allows us to evaluate the robustness of the aggregate model's predictions. This approach may be useful in other areas of accounting research, such as the dynamics of reputation in public accounting, earnings management, the adoption and spread of managerial accounting practices, and the effect of other audit firms' behaviors on auditor independence.

The remainder of the paper begins by describing the assumptions underlying our aggregate-level model. We then analyze the model and develop comparative statics. Next, we introduce the agent-based model and describe a series of numerical experiments examining its comparative statics properties. Finally, we discuss the implications arising from our results, the limitations of this research, and possible future avenues of inquiry.

II. AGGREGATE MODEL ASSUMPTIONS

We model the population of taxpayers in an undefined geographic region; our results could apply to the population of a city, a state, an administrative region of the tax agency, or a country. For simplicity, we assume that the population is constant and consists of three

[4] Kirman (1992) critiques the use of representative agents in analytical models.

[5] Agent-based models are not related to agency theory. Epstein and Axtell (1996) provide an excellent review and a general introduction to agent-based models in the social sciences.

discrete groups of taxpayers. The first group, x_{1t}, is the proportion of *honest* taxpayers in the population.[6] Honest taxpayers comply at time period t. These taxpayers do not consider evasion. They are either habitually compliant (Erard and Feinstein 1994) or they are recent evaders who have become honest as a result of enforcement efforts or social norms. The second group consists of taxpayers who are dissatisfied with the tax system (perhaps as a result of seeing others evade without being punished). These taxpayers are not actively evading, but they might if the perceived benefits of doing so exceed the perceived costs. For this group, evasion is an option, and so we classify them as *susceptible*. The proportion of susceptible taxpayers in the population at time period t is x_{2t}. The third group, x_{3t}, represents the proportion of *evading* taxpayers in the population at time period t. Whether a taxpayer continues to evade depends on both enforcement and the effect of social norms.

Because the entire population is made up of these three classes of taxpayers:

$$x_1 + x_2 + x_3 = 1. \tag{1}$$

We further assume that each proportion is initially positive:

$$x_{10} > 0; \quad x_{20} > 0; \quad x_{30} > 0. \tag{2}$$

We consider any population that satisfies these constraints as valid.

We next describe and model taxpayers' movements among the three classes resulting from the behavior of others, enforcement, and social norms. The model is a system of differential equations.

The Effect of Others' Behavior

Social psychologists have offered several explanations why knowing a tax evader might cause an honest taxpayer to consider evasion. For example, Lerner (1998) suggests that people need to believe the world is just. Consequently, when people observe an unjust event, they may cope by punishing the harm-doer, compensating or blaming the victim, or denying the injustice by reasoning that justice will prevail in the next life. In some cases, such as tax evasion, one might seek justice by engaging in the activity oneself (Spicer and Becker 1980; Tyler 1990).

An alternative explanation for why knowing a tax evader might cause honest taxpayers to consider evasion is that observing others' behavior can affect one's own internalized moral standards. Cooter (1998) argues that people prefer conformity to behavioral standards. When someone violates a standard, they incur a psychological cost—guilt—whether or not others discover the behavioral violation. However, if others of perceived high moral character violate a law, then one's behavioral standard may change. For example, Kaplan and Reckers (1985) provide experimental evidence that subjects were more likely to evade taxes when they observed a taxpayer of perceived high moral character evading.

Consistent with these observations and prior research, in our model as an individual encounters noncompliant taxpayers, he is more likely to consider evasion. We represent this relationship via the following system of differential equations:

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[6] Because we focus on population proportions, our model provides limited insight into the dollar amount of taxes evaded. Evaders can choose activities that permit greater opportunities for evasion, leading to larger dollar amounts evaded than the population proportion would suggest. Other factors may reduce the amount of taxes evaded (e.g., third-party reporting requirements may restrict evasion).

$$\dot{x}_1 = \frac{dx_1}{dt} = -rx_1x_3$$
$$\dot{x}_2 = \frac{dx_2}{dt} = rx_1x_3 \tag{3}$$

where r represents the ‘‘infection rate.’’ For simplicity, we model the rate of change of the honest taxpayer class, x_1, as a proportion of the product of honest taxpayers, x_1, and evading taxpayers, x_3. This is a standard simplification in epidemic models (e.g., Waltman 1974, 2) and implies uniform mixing of the groups and no delay before becoming susceptible.[7]

Modeling the Enforcement Regime

We assume that susceptible and evading taxpayers base their tax-reporting decisions partly on a psychological calculus that trades off the perceived costs and benefits of evasion. A taxpayer's view of these costs and benefits depends on enforcement (e.g., penalties, audit likelihood, audit costs imposed on the taxpayer, the public image of the tax agency, etc.), tax rates and their progressivity, and psychological factors, including framing effects from under- or overwithholding.[8]

We model two effects of enforcement on individual tax compliance behavior. First, we model a flow from the evader class to the honest class. For simplicity, we assume that evaders become compliant after they are audited (akin to a change in driving behavior after receiving a traffic ticket)[9] or when their perceptions regarding the costs and benefits of evasion change, either through experience or changing economic conditions.[10] We also assume that this flow from the evader to the honest class as a result of *enforcement* is proportional (at rate β) to the size of the evader class. This assumption is consistent with the tax agency behaving strategically: as the number of evaders increases, the tax authority increases enforcement efforts and vice versa.

Enforcement also affects ‘‘susceptible’’ (but not ‘‘honest’’) taxpayers' behavior through its effect on the perceived costs of evasion. Recall that susceptible taxpayers' compliance decisions are based on a cost-benefit analysis. We assume that some susceptible taxpayers will perceive that the benefits of evasion exceed the costs of evasion in each period. These taxpayers will evade, creating a flow from x_{2t} to x_{3t}. We assume that this flow is proportional to the size of the susceptible class (at rate α, where α is between 0 and 1). As enforcement increases, the cost of evasion increases, reducing the flow to the evading class. Similarly,

[7] We test the robustness of the uniform mixing assumption and other assumptions regarding the flows among taxpayer classes using our agent-based model.

[8] For example, underwithheld taxpayers tend to perceive payments to the tax agency as losses while overwithheld taxpayers tend to perceive refunds from the tax agency as gains. Because individuals are risk seeking in perceived losses and risk averse in perceived gains (Kahneman and Tversky 1979), under- or overwithholding can affect tax compliance (e.g., Schepanski and Kelsey 1990; White et al. 1993).

[9] Psychological explanations for post-audit compliance include availability (Tversky and Kahneman 1973) and vividness (Nisbett et al. 1976; Plous 1993). This research generally finds that taxpayers judge an audit as more probable if it is vivid or more easily recalled. In addition, taxpayers may experience guilt or stigma if the tax agency discovers their evasion. Combined with availability, it is reasonable to expect such taxpayers to report honestly, at least for a time.

[10] Some taxpayers might flow from the evader class back to the susceptible class. That is, some evaders might not be rehabilitated when they are audited, remaining susceptible rather than becoming honest. Similarly, one might add a flow from the susceptible class back to the honest class to allow for spontaneous recovery from susceptibility. Adding these flows to our model complicates the analysis but the results do not differ significantly from those reported here. Specifically, these alternative models have stable honest and mixed equilibria, they meet our plausibility criteria, and the comparative statics results are qualitatively the same as those reported below.

decreases in enforcement will increase the flow to the evading class. When we add both enforcement-regime effects to the system of differential equations in Equation (3), they become:

$$\begin{aligned} \dot{x}_1 &= \beta x_3 - r x_1 x_3 \\ \dot{x}_2 &= -\alpha x_2 + r x_1 x_3 \\ \dot{x}_3 &= \alpha x_2 - \beta x_3 \end{aligned} \tag{4}$$

Incorporating the Effect of Social Norms

The role of social norms is distinct from the role of others' compliance behavior. A social norm requires consensus about the esteem-worthiness of a behavior and some risk that a member of society will detect behavior that deviates from the norm. Society-at-large imposes sanctions on individuals who are caught violating a social norm.[11] In contrast, as noted above, the effect of others' behavior does not require consensus about the esteem-worthiness of an action, and one's response to the action is independent of detection (e.g., one feels guilty).

Social norms can affect compliance because individuals tend to seek the respect of others (McAdams 1997). Social stigma is associated with tax evasion (Scott and Grasmick 1981) and this stigma (and the view of acceptable compliance behavior) varies from country to country (or region to region). Thus, in our model social norms create an additional flow from the evader class to the honest class (beyond the flow created by enforcement). In total, then, evaders feel pressure to become compliant, as a result of both enforcement and social norms. The function $g(x_1)$ represents this combined effect.

In our model, $g(x_1)$ has two components. The first component is β, the proportional flow from the evader class to the honest class arising from enforcement. The second component of $g(x_1)$ is the flow resulting from social norms. On the basis of research in sociology (Hamblin et al. 1973) and following the approach adopted in the literature (e.g., see Lindbeck et al. 1999, 9), we assume that this second component is s-shaped, so that the overall function $g(x_1)$ is also s-shaped as illustrated in Figure 1. The "s" shape of the second component of $g(x_1)$ suggests that when relatively few people adopt a norm, the effect on others is small. As the consensus about a norm increases, its effect on others increases, initially at an increasing rate and later at a decreasing rate. The effect of a social norm on behavior eventually levels off as the proportion of the population adopting the norm approaches 100 percent. The maximum level of social influence is γ.

Consequently, our full model incorporates:

- the effect of others' behavior, triggering a flow from honesty to susceptibility;
- the effect of a social norm, causing some evaders to become honest each period;
- an enforcement effect triggering a shift from evasion to honesty; and
- an enforcement effect on the flow from susceptibility to evasion.

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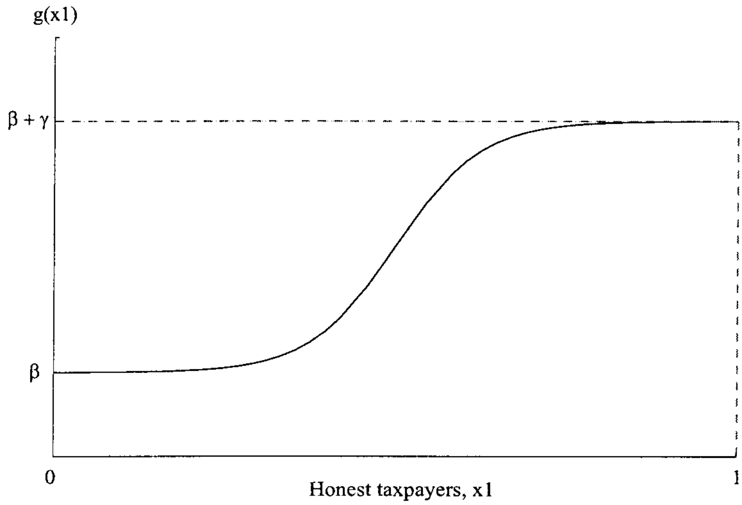
[11] Social norms influence behavior in a number of settings (e.g., see Grasmick and Green [1980] for work in a legal context, and King [2002] for an experimental study showing that professional norms mitigate auditors' propensity to engage in self-serving bias.)

FIGURE 1
The Proportion of Evading Taxpayers Flowing to the Honest Class, $g(x_1)$



The function $g(x_1)$ represents the total proportion of evading taxpayers flowing to the honest class. The flow consists of two components. A proportion of evaders, β, become honest as a result of enforcement efforts. Additional evaders become honest as a result of social norms. The effect of social norms is s-shaped and depends on the proportion of honest taxpayers, x_1. γ is the maximum proportion of evaders that can flow to the honest class as a result of social norms.

The full model is presented below:

$$\begin{aligned}\dot{x}_1 &= g(x_1)x_3 - rx_1x_3\\ \dot{x}_2 &= -\alpha x_2 + rx_1x_3\\ \dot{x}_3 &= \alpha x_2 - g(x_1)x_3\end{aligned} \tag{5}$$

The model in (5) does not rely on traditional game theory, in which the goal is to derive from individuals' rational choices a set of strategy assignments associated with payoffs. The greatest advantage of game theory is that it allows for deduction. The principal shortcomings of a game theory approach are that (1) it cannot explain how the economy changes over time, or how the equilibrium arises, and (2) it focuses on interactions among small numbers of individuals (usually two). In contrast, our dynamic systems analysis stems from an aggregate model that represents the state of the economy by a set of variables, and a system of differential equations describes how these variables change over time. We examine the resulting trajectories. Our approach has much in common with evolutionary game theory focusing on system dynamics (Weibull 1997).[12] However, although our techniques are similar to those used in evolutionary game theory, we do not build on a standard

[12] In accounting, Bloomfield's (1995, 1997) work on the evolution of expectations and its effect on the predictive ability of a Nash equilibrium is in this line of research.

microeconomic foundation. Rather than specifying payoffs, information, and actions for all players, we focus on social-psychological first principles and the consequent evolution of aggregate behavior over time.

The system of equations in (5) describes the way valid populations evolve over time. In our analysis, we begin by establishing that initially valid populations will remain valid as they evolve. Next, we identify equilibrium points that represent balanced populations that are invariant over time. We then evaluate the stability of these equilibria to determine whether one could reasonably expect that they would ever be reached over time by a set of valid starting populations. We complete our analysis by investigating the effect of changes in enforcement regime on the behavior of the system.

III. AGGREGATE MODEL ANALYSIS

Because (5) is a system of differential equations, their solution is a set of functions of time, $x_1 = x_{1t}$, $x_2 = x_{2t}$, and $x_3 = x_{3t}$ for all values of time t greater than some starting point, $t = t_0$. The aim in analyzing the system is to understand the dynamics of the solutions and how the parameters in the model (e.g., r, α, and β) affect these dynamics. We will show that this particular system of equations has at least one stable equilibrium for each class of taxpayer; i.e., x_{1t} tends to a value, x_1^*, as t increases, regardless of the initial value for x_1. This also holds true for x_2 and x_3.

To begin our analysis, we verify that the solutions are consistent with our interpretation. Since x_1, x_2, and x_3 each represent a partition of the population, and together, these three categories are exhaustive, they must sum to 1 at all values of t. That is:

$$x_1(t) + x_2(t) + x_3(t) = 1. \tag{6}$$

To verify that this condition is true, we begin by assuming that Equation (6) holds true for the initial conditions of our model; i.e.:

$$x_1(0) + x_2(0) + x_3(0) = 1. \tag{7}$$

Then, if we differentiate Equation (6) with respect to t, the result is 0 for all values of t, so that the value of Equation (6) equals a constant with respect to time. Hence, if Equation (7) is satisfied initially, then Equation (7) also will be satisfied for all subsequent values of t.

Next, we establish that, given a set of valid initial values for x_1, x_2, and x_3, the values will remain valid as time increases. Using the system of equations in (5), we develop a set of working equations. Through substitution:

$$\begin{aligned} \dot{x}_1 &= [g(x_1) - rx_1]x_3, \\ \dot{x}_3 &= \alpha(1 - x_1 - x_3) - g(x_1)x_3, \\ x_2 &= 1 - x_1 - x_3. \end{aligned} \tag{8}$$

Before proceeding, we introduce the notion of a *vector field*. Just as a function in one variable associates a value or number with each point on the line, a vector field associates a vector with each point in the plane. One can represent a vector field graphically. If a vector field, φ, is defined on some region of the plane, and if we choose a grid of points that covers the region, then we can represent the vector field with arrows drawn from each point, x, where each arrow represents the vector $\varphi(x)$. The arrow's length equals the norm of $\varphi(x)$ and the arrow points in the direction of the vector. One can view the vector field arising from our system of differential equations as the local set of evolutionary laws.

We can represent the system of differential equations in (5) by a vector field that associates the point (x_1, x_2, x_3) in space with the vector $(\dot{x}_1, \dot{x}_2, \dot{x}_3)$. A solution to the equations $(x_1(t), x_2(t), x_3(t))$ traces out an unbroken curve in space as t increases. It has the property that, at each point (x_1, x_2, x_3) on the curve, the tangent to the curve is the value of the vector field at that point.

Recall that a valid population for our model is a point (x_1, x_2, x_3) in R^2 (two-dimensional Euclidean space) where $x_1, x_2, x_3 \geq 0$ and $x_1 + x_2 + x_3 = 1$. The region V containing all valid populations is bounded by $x_1 = 0$, $x_2 = 0$, and $x_3 = 0$. If all initially valid populations in our system remain valid over time, then the vector field $(\dot{x}_1, \dot{x}_2, \dot{x}_3)$ at any point on the boundary of V is directed toward the interior of V. That is, at each point on the boundary of V:

$$(\dot{x}_1, \dot{x}_2, \dot{x}_3) \cdot n \leq 0, \tag{9}$$

where "·" is the dot product operation and n is the outer normal vector (the vector perpendicular to and outside the valid space, V).

On the plane $x_1 = 0$, the outer normal vector is $n = (-1, 0, 0)$ and the dot product is $-\dot{x}_1 = -g(0)x_3 \leq 0$. Similarly, the outer normal vectors for $x_2 = 0$, $x_3 = 0$, and $x_1 + x_2 + x_3 = 1$ are $n = (0, -1, 0)$, $(0, 0, -1)$, and $(1, 1, 1)$, respectively. The resulting dot products are:

$$-\dot{x}_2 = -rx_1x_3 \leq 0,$$

$$-\dot{x}_3 = -\alpha x_2 \leq 0, \text{ and}$$

$$\dot{x}_1 + \dot{x}_2 + \dot{x}_3 = 0.$$

Because all the dot products are less than or equal to 0, we can conclude that any initially valid population will remain valid as time passes. This result is critical to the plausibility of the model.

Equilibrium Populations

Next we consider the behavior of the system of equations as a function of time. We begin by identifying points where the value of each of the differential equations equals 0, indicating no change over time. These points represent equilibria for the three classes of taxpayers. We find the equilibria by solving:

$$\dot{x}_1 = [g(x_1) - rx_1]x_3 = 0, \text{ and} \tag{10}$$

$$\dot{x}_3 = \alpha(1 - x_1 - x_3) - g(x_1)x_3 = 0 \tag{11}$$

for x_1 and x_3. We can then calculate $x_2 = 1 - x_1 - x_3$. Note that $\dot{x}_2 = -\dot{x}_1 - \dot{x}_3$, so that $\dot{x}_1 = \dot{x}_3 = 0$ implies that $\dot{x}_2 = 0$.

Equation (10) is satisfied either when $x_3 = 0$ or when $g(x_1) = rx_1$. Thus, there are two possible cases to investigate. In the first case, when $x_3 = 0$, Equation (11) becomes $0 = \alpha(1 - x_1)$. Since α is a positive number less than or equal to 1, x_1 must equal 1. Thus, an equilibrium may exist in which all taxpayers belong to the honest class.

In the second case, where $g(x_1) = rx_1$, Equation (11) gives $rx_1x_3 + \alpha x_3 = \alpha(1 - x_1)$, so that:

$$x_3 = \frac{1 - x_1}{1 + \frac{r}{\alpha} x_1}.$$

Substituting the solution for x_3 into the working equation for x_2 in Equation (8) and simplifying yields a value for x_2 of:

$$x_2 = \frac{r x_1 (1 - x_1)}{\alpha + r x_1}.$$

The foregoing analysis leads to the following proposition.

Proposition 1: There are two types of equilibrium populations $(\hat{x}_1, \hat{x}_2, \hat{x}_3)$:
(i) A uniformly honest population $(\hat{x}_1, \hat{x}_2, \hat{x}_3) = (1, 0, 0)$;
(ii) A mixed population of the form:

$$(\hat{x}_1, \hat{x}_2, \hat{x}_3) = \left(\bar{x}, \frac{r\bar{x}(1 - \bar{x})}{\alpha + r\bar{x}}, \frac{1 - \bar{x}}{1 + \frac{r}{\alpha} \bar{x}} \right)$$

where $\bar{x}$ satisfies $g(\bar{x}) = r\bar{x}$.

Location of Equilibria

So far, we have established that equilibrium populations, $(\hat{x}_1, \hat{x}_2, \hat{x}_3)$, may exist and that these populations may be either uniformly honest or mixed. To determine the location of the mixed equilibria, we must identify the values of x_1 where $g(x_1) = rx_1$. In addition, each equilibrium must satisfy the constraints $\hat{x}_1, \hat{x}_2, \hat{x}_3 \geq 0$ and $\hat{x}_1 + \hat{x}_2 + \hat{x}_3 = 1$. Assume that $(\hat{x}_1, \hat{x}_2, \hat{x}_3)$ is a type (ii) equilibrium with $0 \leq \hat{x}_1 \leq 1$. Then $1 - \hat{x}_1 \geq 0$. Because α and r are positive, $1 + (r/\alpha)\, \hat{x}_1 \geq 0$. It follows that $\hat{x}_3 \geq 0$ (see the formula for $\hat{x}_3$). Note that:

$$\frac{1 - \hat{x}_1}{1 + \frac{r}{\alpha} \hat{x}_1} \leq \frac{1}{1 + \frac{r}{\alpha} \hat{x}_1} \leq 1. \qquad (12)$$

So $\hat{x}_3 \leq 1$. Now, consider the sum:

$$\hat{x}_1 + \hat{x}_3 = \hat{x}_1 + \frac{1 - \hat{x}_1}{1 + \frac{r}{\alpha} \hat{x}_1}$$

$$= \frac{1 + \frac{r}{\alpha} \hat{x}_1^2}{1 + \frac{r}{\alpha} \hat{x}_1}.$$

By hypothesis, each term is not negative, so the sum is not negative. For $\hat{x}_1 \in [0, 1]$, $\hat{x}_1^2 \leq \hat{x}_1$, so:

$$\frac{1 + \frac{r}{\alpha}\hat{x}_1^2}{1 + \frac{r}{\alpha}\hat{x}_1} \leq \frac{1 + \frac{r}{\alpha}\hat{x}_1}{1 + \frac{r}{\alpha}\hat{x}_1} = 1. \tag{13}$$

We can therefore conclude that $\hat{x}_1 \in [0, 1]$ implies that $\hat{x}_1 + \hat{x}_3 \in [0, 1]$. It follows that $\hat{x}_2 \in [0, 1]$. This reduces the search for type (ii) equilibria to finding all $\overline{x}$ satisfying $g(\overline{x}) = r\overline{x}$ and $\overline{x}_1 \in [0, 1]$, leading to the following proposition.

Proposition 2: An equilibrium $(\hat{x}_1, \hat{x}_2, \hat{x}_3)$ of type (ii) is valid if and only if $0 \leq \hat{x}_1 \leq 1$ and $g(\hat{x}_1) = r\hat{x}_1$.

The proportion of honest taxpayers is in equilibrium when the proportion of taxpayers flowing into the honest class, $g(\hat{x}_1)x_3$, equals the taxpayers flowing out of the honest class, $r\hat{x}_1x_3$. When the honest proportion is in equilibrium, then both the susceptible and evading classes must also be in equilibrium because the flows among the classes are constant.

Understanding the Dynamics of the System

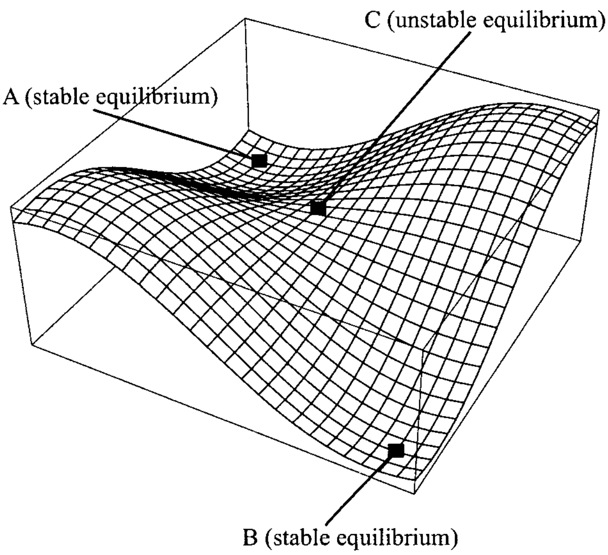
We have established that equilibria exist in the system, but we must address two additional issues to determine whether and under what conditions the equilibria will be reached. First, we must determine whether *basins of attraction* exist (i.e., whether the equilibria attract nearby points). Second, if basins of attraction do exist, we must determine whether they extend to all valid starting populations. If this second condition is satisfied, then all valid populations will eventually reach one of our equilibria, and the dynamics of the system will be fully specified.

An equilibrium point that attracts nearby points over time is *stable*. We define an equilibrium to be stable if there is a neighborhood N of (x^*, y^*) such that if (x_t, y_t) is in the neighborhood N, then $\lim_{t\to\infty} (x_t, y_t) = (x^*, y^*)$ (Arnol'd 1991, 156). For an unstable equilibrium, within even a tiny neighborhood N of the point, some (x_0, y_0) exists in N and some neighborhood U of the equilibrium such that (x_t, y_t) is never permanently in U.

More intuitively, imagine an empty pool with a saddle-shaped bottom, as in Figure 2. If one were to drop a ball in the pool, it would likely roll toward one of the deep ends, eventually coming to rest at one of the two deepest points (labeled A and B, respectively). These points at which the ball stops moving represent stable equilibria. They are stable in that, if the ball is in the point's "neighborhood," (i.e., the sloped area that surrounds the point), then the ball will be attracted to that particular point. Alternatively, the ball could be carefully balanced at point C, the lowest point in the shallow section (i.e., not on either slope to the deep ends), and remain at rest. This point is also an equilibrium; however, it is unstable. A small movement of the ball away from this point would cause it to roll to one of the deep ends, eventually coming to rest at A or B.

We investigate the stability properties of our system of differential equations to gain insight into the behavior of the system over time. For example, given a stable equilibrium and selected initial conditions close enough to this point, the solution path to a system of differential equations will tend toward and eventually reach equilibrium. Alternatively, a selected initial condition may lie near an unstable equilibrium. In this case, the system may

FIGURE 2
Basins of Attraction and Stable vs. Unstable Equilibria



This surface illustrates stable and unstable equilibria. A ball placed on the surface near point A (or B) will roll toward the point and eventually come to rest there. Thus, A and B are stable equilibria. A ball placed near C may initially roll toward C, but it will not come to rest there (although a ball carefully balanced at C will remain at rest). C is an unstable equilibrium.

initially tend toward, but not stabilize at, the critical point. Consequently, there would be no reason to expect the equilibrium to obtain.

To ascertain stability in our system, we evaluate a linear approximation of the system near an equilibrium. For the case of an honest equilibrium, our analysis (formally presented in the Appendix) leads to the following proposition regarding stability.

Proposition 3: Assume that $g(x)$ has a convergent Taylor series expansion near $x = 1$ and $g(1) > 0$. The equilibrium $(1, 0, 0)$ is stable if and only if $r < g(1)$.

Proof. See the Appendix.

Proposition 3 is intuitive in the neighborhood of an honest equilibrium, where $g(1)$ captures the effect of social norms and enforcement on the conversion of evaders into honest taxpayers. When the flow of honest taxpayers into the susceptible class (via the infection rate, r) is less than the flow from the evading class into the honest class (via a value near $g(1)$), then all taxpayers will eventually become honest.

An analysis of the stability of the mixed equilibrium leads to the following proposition.

Proposition 4: Assume that for any $\bar{x} \in [0, 1]$, $g(x)$ has a convergent Taylor series near $\bar{x}$ and $g(\bar{x}) \geq 0$. Let $(\hat{x}_1, \hat{x}_2, \hat{x}_3)$ be a mixed equilibrium. The equilibrium is stable if and only if $r > g'(\hat{x}_1)$.

Proof. See the Appendix.

To understand the intuition underlying Proposition 4, recall that, by Proposition 1, the proportion x of honest taxpayers is in equilibrium when the proportion rx of infected taxpayers reduces the honest population at the same rate that social forces and enforcement (g(x)) increase it. This equilibrium will be stable if proportions below x are driven upward and proportions above x are driven downward. This happens when $g(x) > rx$ for points below x and when $g(x) < rx$ for points above x. These inequalities hold only if $g'(x) < r$.[13]

Determining the Domain of Stability

Having evaluated the conditions under which equilibria are stable, we turn our attention to the second issue: whether and under what conditions the domain of stability encompasses the entire set of valid populations (global stability). In evaluating nonlinear dynamic systems, one can rarely derive equations for the domain of stability (Lyapunov functions), and unfortunately this holds true for our case as well. However, using a combination of graphics, computational analysis, and an appeal to the Poincaré-Bendixson theorem (P-B theorem), we provide evidence that all valid populations will eventually reach one of the stable equilibrium points.

The P-B theorem applies when the vector field is planar and bounded, and equilibria are isolated. When these conditions are met (as in our system), either (1) the limit set (the range of points possible in a neighborhood as a result of the system dynamics) or a subset of the limit set is an equilibrium, or (2) the limit set is a periodic orbit. To ascertain which of these outcomes applies to our system, we turn to a qualitative evaluation of the system.

Recall from Proposition 1 that honest equilibria always exist, and from Proposition 2 that mixed equilibria exist only where x_1 is between 0 and 1 and the line rx_1 and the function $g(x_1)$ intersect. In Panel A of Figure 3, we provide an example in which three equilibria exist: two mixed equilibria at points a and b and an honest equilibrium at point c.

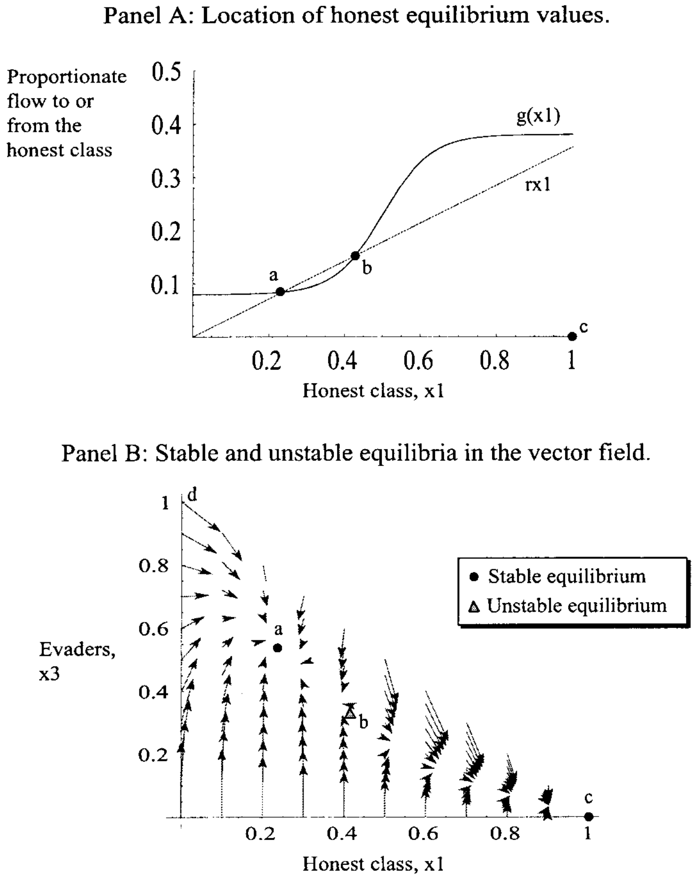
We can evaluate the stability of these equilibria using the results of Propositions 3 and 4. According to Proposition 3, a completely honest equilibrium is stable if and only if $r < g(1)$. That is, the honest equilibrium is stable when the flow of honest taxpayers into the susceptible class is less than the flow of evaders into the honest class. This condition is satisfied in Panel A of Figure 3, so the honest equilibrium c is stable.

For a mixed equilibrium to be stable, populations below the equilibrium must be driven upward toward the equilibrium and populations above the equilibrium must be driven downward toward the equilibrium. This happens when $g(x) > rx$ for points below x and when $g(x) < rx$ for points above x. According to Proposition 4, these inequalities hold only when the slope of the line rx_1 exceeds the slope of $g(x_1)$ at the intersection point (i.e., when the proportions below the equilibrium are driven upward and proportions above the equilibrium are driven downward). This condition is met at a, but not at b. Thus, a is a stable equilibrium value for the honest population proportion (x_1) and b is unstable.

Having identified the honest-population proportions (x_1) in our three equilibria in the example, we can use the results of Proposition 1 to compute the proportion of evaders (x_3) and susceptible taxpayers (x_2) for each mixed equilibrium. We graphically represent these equilibria in the (x_1, x_3) plane in Panel B of Figure 3. The black dots represent stable equilibria (at a and c) and the gray triangle (at b) represents an unstable equilibrium.

[13] We thank one of our anonymous reviewers for this insight.

FIGURE 3
The Location of Equilibria, Their Stability, and the Movement of Out-of-Equilibrium Populations over Time



Panel A illustrates how we identify equilibrium values. Mixed equilibria exist when the number of evading taxpayers moving to the honest class equals the number of honest taxpayers moving to the susceptible class (when $g(x_1)$ equals rx_1, at points a and b). An honest equilibrium always exists. The honest equilibrium is stable when the flow out of the honest class is less than the flow into the honest class (when the value of r is less than the value of g(1)). In Panel A, this condition is satisfied for point c. Mixed equilibria are stable if the slope of rx_1 exceeds the slope of $g(x_1)$ at the intersection point. This condition is met at a, but not at b.

Panel B shows the equilibrium values on the (x_1, x_3) plane, together with the associated vector field. The arrows in the vector field illustrate the movement of points over time toward the stable equilibria, a and c. This figure illustrates only one of several possible relations between rx_1 and $g(x_1)$.

Panel B of Figure 3 also uses arrows to illustrate the vector field associated with the system. The vector field here is a two-dimensional representation of a saddle-shaped surface similar to that depicted in Figure 2. We constructed the vector field computationally by determining the dynamic solution to the system for numerous starting populations and representing them graphically as arrows.[14] The direction of each arrow shows the path the system takes over time from the arrow's origin. The length of the arrow represents the slope of the surface (longer arrows are equivalent to a steeper slope). To illustrate, given an initial population in which everyone evades (at d in Panel B of Figure 3), the population would move to point a over time and then remain there (in equilibrium). We can observe the dynamics by following the path suggested by the arrows, beginning at d and ending at a. From the figure (and the P-B theorem), one can see that valid populations (except for the unstable equilibrium at b) appear to move toward either a or c. Hence, our analysis suggests that the limit set in the system is either a, b, or c (i.e., the system appears to exhibit global stability). We observe no evidence of a periodic orbit in the vector field.

Figure 3 depicts only one possible relation between the line rx_1 and the function $g(x_1)$. We evaluated 11 possible relations that arise from combinations of differently sloped lines rx_1 and differently shaped $g(x_1)$. These 11 relations are exhaustive in that they describe all types of possible equilibria and the related dynamics that could occur in our system. In every case, either one or two stable equilibria exist and we observe no evidence of a periodic orbit.

The Effect of Enforcement on Evasion

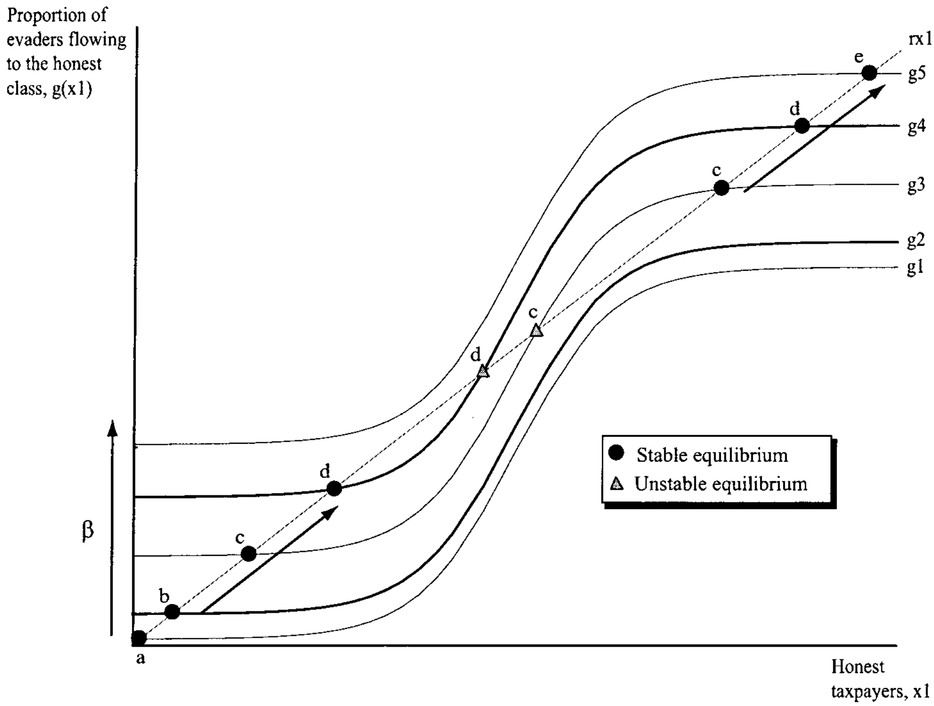
Enforcement affects the movement of susceptible taxpayers to the evading class (α) and the movement of evaders to the honest class (β). To better understand the effect of enforcement in our model, we begin by examining benchmark cases for these two flows. For the case in which $\alpha = 0$, susceptible taxpayers remain susceptible and never become evaders (presumably because of draconian enforcement), by the system of differential equations in (5). As long as at least some evaders are initially present in the system, there is a flow out of the evading population and into the honest population, until, in the limit, all taxpayers comply with the law (being either honest or susceptible). This result obtains even if there are no honest taxpayers initially, because $g(0) = \beta > 0$. In equilibrium, as α decreases (and perceived enforcement severity increases), by Proposition 1, the proportion of evaders decreases at an increasing rate and the proportion of susceptible taxpayers increases.

An analysis of β is more complex. In an initial condition in which there are no honest taxpayers ($x_1 = 0$) and no enforcement efforts ($\beta = 0$), $g(x_1)$ has no effect. There is no flow from the evading class to the honest class. In the limit, the entire population evades (assuming $\alpha > 0$). However, if some proportion of the initial population is honest ($x_1 > 0$), then some evaders become honest in each period, as a result of social norms. Because β affects only the intercept and not the shape of $g(x_1)$, the system dynamics and the limit set depend on the relation between r and $g(x_1)$, as noted previously.

Figure 4 illustrates the effect of increasing the enforcement-related flow from the evading class to the honest class, β, on the equilibrium values for x_1. At low levels of β (on curves g1 and g2), the proportion of honest taxpayers is low (at stable equilibrium points a and b). As β increases, the curve moves from g2 to g3. At some point between these curves, $g(x_1)$ will be tangent to rx_1. Beyond this point, the line and the curve will intersect in three places. These intersection points, c, are identified on g3. The high and the low points are stable equilibria and the middle point is an unstable equilibrium. The unstable

[14] The qualitative aspects of the vector field obtain for any g(x) satisfying the relations in Panel A. For computation purposes, we use the function $g(x) = \beta + (4a/d(1 + e^{db}))[(e^{db} - e^{-d(x-b)})/(1 + e^{-d(x-b)})]$ where the equality $\gamma/a = 4(1 - e^{-d})/(d(1 + e^{-db})(1 + e^{-d(1-b)}))$ is satisfied if and only if $a > \gamma > 0$. A proof showing that this equation satisfies the criteria required for g(x) is available from the authors.

FIGURE 4
The Effect of Increasing the Enforcement-Driven Flow from the Evading Class to the Honest Class, β, on the Equilibrium Proportion of Honest Taxpayers, x_1



As the enforcement-driven flow from the evading class to the honest class, (β), increases, the total proportion of evading taxpayers moving to the honest class, $g(x_1)$, moves upward from curve g1 to g5. When β is small (g1), only one stable equilibrium exists at a. In this equilibrium, only a few taxpayers are honest. Increasing enforcement to g2, the proportion of honest taxpayers in equilibrium increases to b. As enforcement continues to increase (g3), three equilibria exist, all denoted by c. The high and low intersection points are stable equilibria and the middle point is unstable. At a higher level of enforcement (g4), the proportion of honest taxpayers increases under both stable equilibria but decreases in the unstable equilibrium (all denoted by d). At high levels of enforcement (g5), only one stable equilibrium exists, denoted by e. Most taxpayers in this equilibrium are honest.

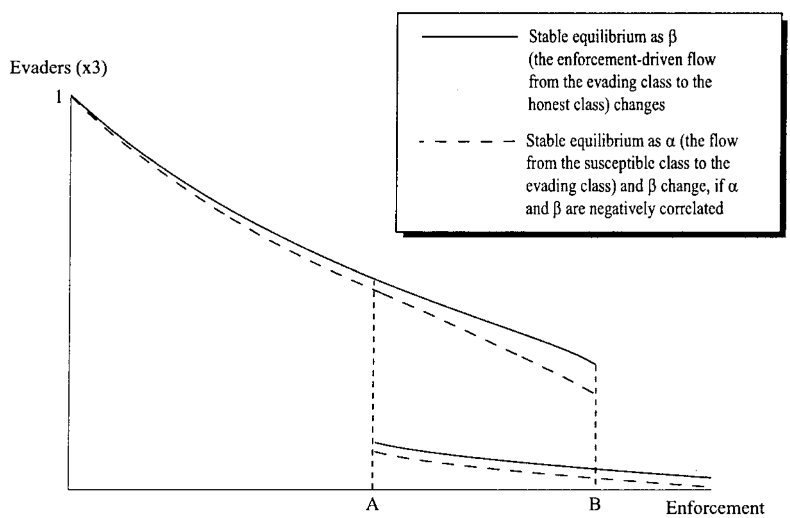
equilibrium plays an important role in determining the dynamics of the system. The vector field in Panel B of Figure 3 indicates that the system is characterized by a saddle-shaped surface. The unstable equilibrium is on the saddle, so that out-of-equilibrium populations above the unstable equilibrium will move to the high equilibrium point, while out-of-equilibrium populations below the unstable equilibrium will move to the low equilibrium point over time. As β continues to increase and $g(x_1)$ moves to g4, the proportion of honest taxpayers increases in the two stable equilibria and decreases in the unstable equilibrium (at d). As the unstable equilibrium decreases, out-of-equilibrium populations that would have moved to the lower stable equilibrium under g3 begin, under the greater enforcement depicted in g4, to move to the upper stable equilibrium instead. Finally, as $g(x_1)$ moves from g4 toward g5, the line and curve will again reach a point where they are tangent. Beyond this point, they intersect only once. This intersection occurs on g5 at e. In this (stable) equilibrium, most taxpayers are honest. This figure also highlights the importance of the s-shaped function describing the flow of taxpayers from the evading class to the honest class; multiple equilibria arise as a result of the nonlinear effect of social norms at some levels of β.

Figure 5 summarizes our analysis.[15] As enforcement increases, β increases. The solid curves show how an increase in β affects the evading proportion in equilibrium. Based on our analysis of Figure 4 (viz., curves g1 and g2), at low levels of enforcement (below A in Figure 5), the proportion of honest taxpayers is small in equilibrium. By Proposition 1, x_1 and x_3 are negatively correlated, so the proportion of evaders is relatively high in these equilibria. A and B in Figure 5 correspond to the values of β where rx_1 and $g(x_1)$ are tangent (between curves g2 and g3 and again between curves g4 and g5 in Figure 4). At intermediate levels of enforcement (between A and B), two stable equilibria exist, admitting the possibility of either a mostly evading population or a mostly compliant population. This result follows from curves g3 and g4 in Figure 4. At sufficiently high levels of enforcement (above B), the system returns to a single, mostly honest equilibrium.[16] Curve g5 in Figure 4 illustrates this result.

[15] We constructed Figure 5 from a numerical analysis, using the equation in footnote 14 as g(x).

[16] For some parameterizations of rx_1 and $g(x_1)$, the lower curve in Figure 5 reaches 0, or a completely honest population.

FIGURE 5
The Effect of Enforcement on Taxpayer Evasion



The solid curves in the figure show stable equilibrium values as enforcement changes, assuming that enforcement affects only β (the flow from the evading to honest class). The dashed curves show stable equilibrium values if enforcement affects both β and α (the flow from the susceptible to the evading class). Below A and above B, there is a unique equilibrium value for each level of enforcement. Between A and B, two stable equilibria exist. If a population in equilibrium at point A on the lower curve experiences a decrease in enforcement, then it will move to a new equilibrium along the upper curve. Similarly, populations at point B on the upper curve will move to a new equilibrium on the lower curve if the tax agency increases enforcement.

An increase in enforcement also decreases the proportion of susceptible taxpayers in our model who become evaders (α). By Proposition 1, when α decreases, the number of evaders decreases at an increasing rate. This relation holds for all stable, mixed equilibria. Thus, α reinforces β, further increasing the responsiveness of the population to enforcement efforts. The dashed curves in Figure 5 illustrate the joint effect of enforcement through both α (the reduced flow from the susceptible to the evading class) and β (the increased flow from the evading to the honest class).

We now describe the dynamic behavior of initially out-of-equilibrium populations. Because our analysis suggests global stability, at low and high levels of enforcement, all populations will move toward and eventually reach the unique equilibrium value on the curve. As noted previously, the unstable equilibrium value determines which equilibrium a population will reach. Populations with honest proportions smaller than the unstable equilibrium value will move to the noncompliant equilibrium, while populations with honest proportions greater than the unstable equilibrium will move to the compliant equilibrium.

We can make several interesting observations using Figure 5. Consider a population that has reached a unique equilibrium value on the upper curve, where enforcement is below A and evasion is relatively widespread. If we repeatedly increase enforcement in very small increments and allow the population to adjust to a new equilibrium after each change, it will follow the path from left to right suggested by the upper curve to point B. These increases in enforcement reduce evasion (evidenced by the downward slope of the curve), but a large proportion of the population will still evade. When enforcement exceeds B by an infinitesimal amount, the population will move to the lower curve in equilibrium, resulting in a mostly compliant regime. Thus, a small increase in enforcement leads to a dramatic shift from aggregate noncompliance to aggregate compliance. From a policy perspective, this result suggests that, in regions or time periods where evasion is widespread, increased enforcement may initially have a limited effect on compliance. At some point (B in our figure), however, enforcement can trigger a more profound change in the population's behavior, leading to a mostly compliant regime. Once this occurs (and the population reaches an equilibrium on the lower curve), the tax agency can slowly relax enforcement efforts (down to A in the figure) with little change in the number of evaders. The population will follow the equilibrium suggested by the lower curve if we allow it to adjust to a new equilibrium after each small change. However, an infinitesimal reduction in enforcement below A will again trigger a dramatic change. The population will move to a new equilibrium on the upper curve, where many more taxpayers evade. Attempts to remedy the resulting evasion "epidemic" by increasing enforcement to a point between A and B, which was previously sufficient to maintain compliance, would be largely unsuccessful because the population would again follow the upper curve until enforcement reaches B.

IV. THE AGENT-BASED MODEL

Our aggregate-level model has several shortcomings. First, to maintain tractability, we must make unrealistic assumptions regarding movement among classes of taxpayers. For example, we assume that the flows between classes are in constant proportions (even though there are likely to be random fluctuations in the real world, introduced by randomness in audit frequency and individual behavior). Second, we assume that social norms have an s-shaped effect on the movement from the evading class to the honest class. Third, the aggregate-level model does not explicitly account for micro-level taxpayer behavior. To address these limitations, we supplement our analysis with a micro-level agent-based computational model.

Background

Agent-based modeling has roots in biology (e.g., Crist and Haefner 1994; Haefner and Crist 1994) and is becoming a more popular method to address social sciences issues[17] as advances in computing technology have made these models feasible.[18] Agent-based models explicitly characterize individual agents as collections of internal states and endowments (e.g., knowledge and assets). Some agent states and endowments are fixed for the life of the agent, whereas others change as a result of interactions with other agents or the environment. In addition to agents, agent-based models also characterize the environment in which agents interact. An environment might represent a landscape or lattice of resource-bearing sites in ecological or anthropological research, a set of institutional (market) rules in economics, or a communication network in a society (in sociology). Agents can either interact with their environments (e.g., travel through their environments to collect resources) or with each other through their environments (e.g., engage in trade) via a set of behavioral and institutional rules.

After defining agents, their behaviors and endowments, and the environment, we allow the model to evolve. Agent-based models focus on aggregate-level behaviors that emerge endogenously from agent interaction. In particular, researchers use agent-based models to identify micro-level characterizations that are sufficient to generate macroscopic social structures or collective (aggregate) behaviors. Put another way, agent-based models grow collective behaviors ‘‘from the bottom up’’ (Epstein and Axtell 1996).

Agent-based models have both advantages and disadvantages (see Axtell [2000] for a more detailed discussion). On the positive side, it is easy to introduce heterogeneity and avoid an appeal to a representative agent, and it is easy to limit agent rationality. In addition, in agent-based models, there is no need to focus on equilibria because the entire dynamic history of the process is available to the researcher. Finally, agent-based models allow investigation of the effects of physical location and social networks.

The most significant disadvantage of agent-based models relative to traditional mathematical models relates to robustness of results. Although each run of an agent-based model amounts to a sufficiency theorem, the model provides no information as to whether the set of assumptions is necessary for the observed results. As Axtell (2000, 6) noted, agent-based models can provide only limited insight into the question: ‘‘Given that agent model A yields result R, how much change in A is necessary in order for R to no longer obtain?’’ Analytical models can usually formally answer this question, but agent-based models can only address it via multiple runs, with systematic variation in initial conditions and parameters. Even then, there are limits to the number of model variations that we can examine (although the continued increase in computing power reduces this limitation).

Creating a Society of Taxpayers

Our agent-based model is based on the framework developed by Gaylord and Davis (1999), which provides a general approach for creating nonspatial agent-based models using

[17] Schelling (1969, 1971, 1978) pioneered the use of agent-based models in social science research, using the method to study the formation of segregated neighborhoods. More recently, agent-based models have provided new insights in economics (e.g., see Marimon et al. 1990; Albin and Foley 1992; Marengo and Tordjman 1996; Arthur et al. 1997; Weisbuch et al. 1998), sociology and social psychology (Carley 1991; Glance and Huberman 1993; Danielson 1996; Page and Hong 1996), anthropology (e.g., Kohler et al. 1999), and political science (e.g., Axelrod 1997; Kollman et al. 1997).

[18] For example, the small experiment reported in this paper took approximately 720 hours of processing time on a 550 MHz personal computer.

Mathematica. We model a society of taxpayers, represented by a matrix (in which each row represents the attributes of a single taxpayer) and a set of transformation rules (representing behavioral rules and the environment through which taxpayers interact). Our model evolves by iteratively mapping the transformation rules to the matrix.[19]

Each taxpayer in our model has several attributes, beginning with a unique name. In addition, since taxpayers update their beliefs about enforcement severity and social norms based on the experiences and behaviors of their acquaintances, we specify a randomly determined set of acquaintances for each taxpayer. The model assumes that acquaintance relationships are symmetric (i.e., if taxpayer 1 knows taxpayer 2, then taxpayer 2 also knows taxpayer 1). Each taxpayer's list of acquaintances is fixed.

In the initial condition for our society, we randomly assign one of two behaviors (honest or evading) to each taxpayer. As a society evolves, the taxpayer's behavior may change as a function of his characteristics and his acquaintances' beliefs and behaviors. Honest taxpayers may become susceptible to evasion by viewing the reporting behavior of a randomly chosen acquaintance. If the randomly chosen acquaintance is an evader, then the taxpayer becomes susceptible with some probability (randomly and independently determined for each taxpayer). This probability represents the agent's susceptibility to "infection," analogous to r in the aggregate model, except that the level of susceptibility is a taxpayer characteristic rather than an aggregate flow.

In our model, the tax agency can imperfectly distinguish evaders from compliant taxpayers, so the agency audits evaders with a higher probability. Taxpayers form a belief regarding the severity of enforcement by observing the frequency with which their acquaintances are audited.[20] Susceptible taxpayers will choose to evade if their perception regarding enforcement severity is below a randomly determined threshold[21] (determined independently for each taxpayer). This agent-level threshold captures the effect of the taxpayer's risk preferences and the expected benefits of evasion, which we assume are heterogeneous over our population.

In addition to forming a belief about the severity of the enforcement regime, taxpayers in the agent-based model also form a belief regarding the extent to which honesty represents a social norm. We implement this belief as the proportion of honest taxpayers in each agent's list of acquaintances. As in our aggregate model, the social norm can cause an evader to become honest. In our agent-based model, evading taxpayers convert to honest behavior if (1) they are audited, (2) their perception of enforcement severity equals or exceeds their threshold, or (3) their perception regarding honesty as a social norm exceeds a randomly determined threshold (determined independently for each taxpayer). Table 1 summarizes the agent-based model.

We designed the agent-based model to relax our earlier simplifying assumptions regarding the movement between classes of taxpayers. Consider, for example, our aggregate model's assumption that flows between classes are uniform and proportional. In each time step of our agent-based model, even if every honest taxpayer knows an evader, movement

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[19] A detailed description of the *Mathematica* code for our model is available from the authors.

[20] Although the base rate of tax audits is common knowledge in some countries (e.g., the U.S. press reports overall audit rates), individuals do not respond normatively to such information. For example, they tend to underweight base rates (Kahneman and Tversky 1973). In addition, other factors affect individuals' likelihood judgments, including the ease with which they can recall occurrences of the event (Tversky and Kahneman 1973). Our assumption that taxpayers form their beliefs on the basis of their own experience and on the basis of their acquaintances' experiences is consistent with people's tendency to underweight base rates and base judgments of likelihood on ease of recall.

[21] For simplicity, we draw all randomly determined values from a uniform distribution.

TABLE 1
Features of Our Agent-Based Model: Taxpayer Attributes and Schedule of Taxpayer Actions

Panel A: Taxpayer Attributes

1. Unique identification number	Each taxpayer's identification number, used as a reference when accessing acquaintance lists.
2. Acquaintance list	A randomly determined, unchanging list of other taxpayers that a given taxpayer consults when updating beliefs regarding enforcement and social norms.
3. Current reporting behavior	The taxpayer's reporting behavior in the current period (honest, susceptible, or evading).
4. Perception of enforcement	The mean audit rate among the taxpayer's acquaintances in the previous period.
5. Perceived social norm	The proportion of the taxpayer's acquaintances who were honest in the previous period.
6. Enforcement threshold	A randomly determined number between 0 and 1 (determined independently for each taxpayer at time step 0). A susceptible taxpayer will evade if her perception regarding the severity of enforcement (attribute 4) is less than this number.
7. Susceptibility parameter	The probability with which a taxpayer will become susceptible if he observes an acquaintance evade. This parameter is a random number between 0 and 1, determined independently for each taxpayer at time step 0.
8. Audit indicator	Indicates whether the taxpayer was audited in the previous time step.
9. Norm threshold	A randomly determined number between 0 and 1 (determined independently for each taxpayer at time step 0). An evading taxpayer will become honest if her perception regarding social norms (attribute 5) is more than this number.

Panel B: Schedule of Taxpayer Actions in Each Time Step

1. Each taxpayer begins a time step by updating her reporting behavior on the basis of a set of rules: (a) honest taxpayers become susceptible with some probability if a randomly selected acquaintance evades; (b) susceptible taxpayers become evaders if perceived enforcement severity is less than their thresholds; (c) evaders become honest if perceived enforcement severity exceeds their thresholds, or if the perceived social norm exceeds their thresholds.
2. The tax agency randomly selects taxpayers to audit, where the probability of auditing an evader is higher than the probability of auditing others. The audit indicator for each taxpayer (attribute 8) is updated to reflect this action. Taxpayers who are not honest become honest upon audit.
3. Each taxpayer updates her perception of social norms (attribute 5) by polling the reporting behavior of taxpayers in her acquaintance list.
4. Each taxpayer updates her perception of the severity of the enforcement regime (attribute 4) by ascertaining the audit frequency of taxpayers in her acquaintance list.

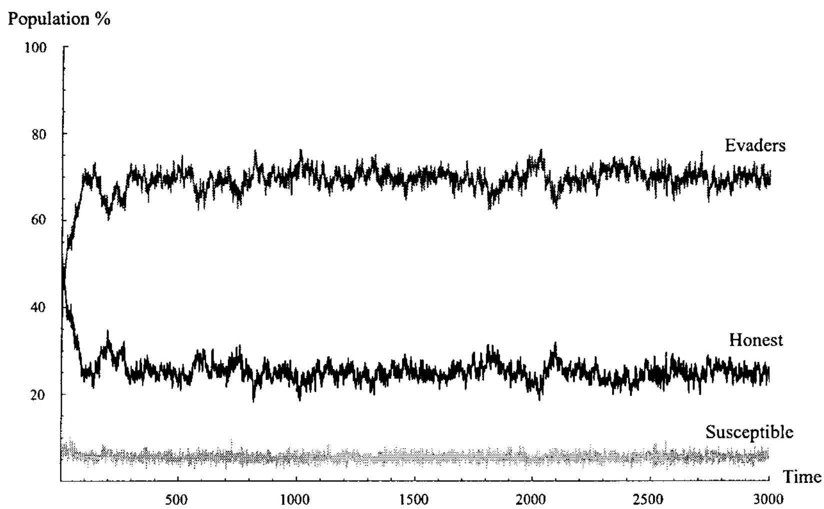
to the susceptible class is *randomly* determined on the basis of each agent's susceptibility. Therefore, random proportions of honest taxpayers become susceptible, and it is possible that all agents could either become susceptible or remain honest. Similar randomness arises in other flows between taxpayer classes. Second, by modeling the decisions of individual agents, we avoid the assumption that social norms have an s-shaped effect on movement from the evading class to the honest class. That is, we describe the model at the level of individual agents, with no attempt to mathematically aggregate individual agent behaviors. Consequently, there is no guarantee that our earlier predictions that hinged on the s-shape of g(x) (i.e., on the nonlinear effect of social norms) will hold in our agent-based model.

Agent-Based Analysis and Results

In light of our aggregate model results, two properties of the agent-based model are of particular interest. First, do stable equilibria exist? Second, do the comparative static properties of our agent-based model correspond with the predictions in the aggregate model?

To solve the agent-based model, we iterate it forward through time, observing the aggregate dynamics. A typical time series illustrating changes in reporting behavior appears in Figure 6. The population rapidly moves from its initial condition (50 percent evading) toward a value of about 70 percent evading. Then, for the remaining periods, the proportion of the population evading fluctuates around 70 percent. Although this behavior does not precisely match our stability prediction, it is consistent with an alternative notion of stability in which an equilibrium is considered stable if points in the neighborhood of the equilibrium

FIGURE 6
Agent-Based Model: Typical Time-Series Evolution (over 3,000 time steps) of a Taxpayer Population That Was Initially 50 Percent Compliant



Taxpayer compliance behavior aggregated from our agent-based model does not reach a constant state as predicted in the aggregate model. Instead, compliance behavior varies randomly around specific values (roughly 70 percent evaders, 25 percent honest, and 5 percent susceptible). This randomness arises because the agent-based model relaxes the assumption of constant flows among classes of taxpayers.

tend to remain in the neighborhood over time (Lyapunov stability). Thus, our earlier inference that equilibria tend to attract nearby points appears to generalize to the agent-based model.[22]

To examine whether the comparative static properties of our aggregate model generalize to the agent-based model, we created nine matched pairs of taxpayer societies. The members of each pair were identical, except for the starting proportion of tax evaders, which we manipulated within pairs at 10 percent and 50 percent. In addition, we manipulated the evader audit likelihood from 0.002 to 0.030 in 0.002 increments for each of the 18 societies.[23] Each society had 500 taxpayers and each taxpayer initially had five acquaintances (after we applied symmetry, the average number of acquaintances was approximately nine). We allowed each society to evolve at each audit rate for 2,000 time steps.

Figure 7 depicts the proportion of evaders in the final time step for each pair of starting populations, by audit rate. For the most part, the graphs resemble Figure 5, corroborating our earlier aggregate model analysis. A sudden increase in compliance is evident in noncompliant populations as enforcement increases (between 20 percent and 60 percent of the population changes from evading to complying in response to a 0.002 increase in audit rates). Likewise, compliant populations are compliant unless enforcement is sufficiently lax, at which point there is a sudden transition to noncompliance. This happens in all but one of the graphs in Figure 7, with increases in evasion ranging from 35 percent to more than 70 percent in response to a 0.002 decrease in audit rates. The figure highlights these sudden changes in equilibrium compliance with darkened segments on the audit-rate axis.[24] The results from our agent-based model differ from Figure 5 in one important way. Rather than observing a ''mostly honest'' population at sufficiently high audit rates, we observe total honesty. Our aggregate model predicts this result if the rate at which taxpayers become susceptible, r, is less than $g(1)$ when the enforcement-related flow from the evading class to the honest class, β, equals 0.

In summary, our agent-based model supports the predictions from our aggregate-level analysis. We find evidence supporting the existence of multiple equilibria at some enforcement levels (conditioned on the compliance level of starting populations). In addition, the effect of changing enforcement severity in our agent-based model is consistent with predictions from the aggregate model. At critical points, small changes in enforcement can lead to large changes in compliance.

V. CONCLUSIONS

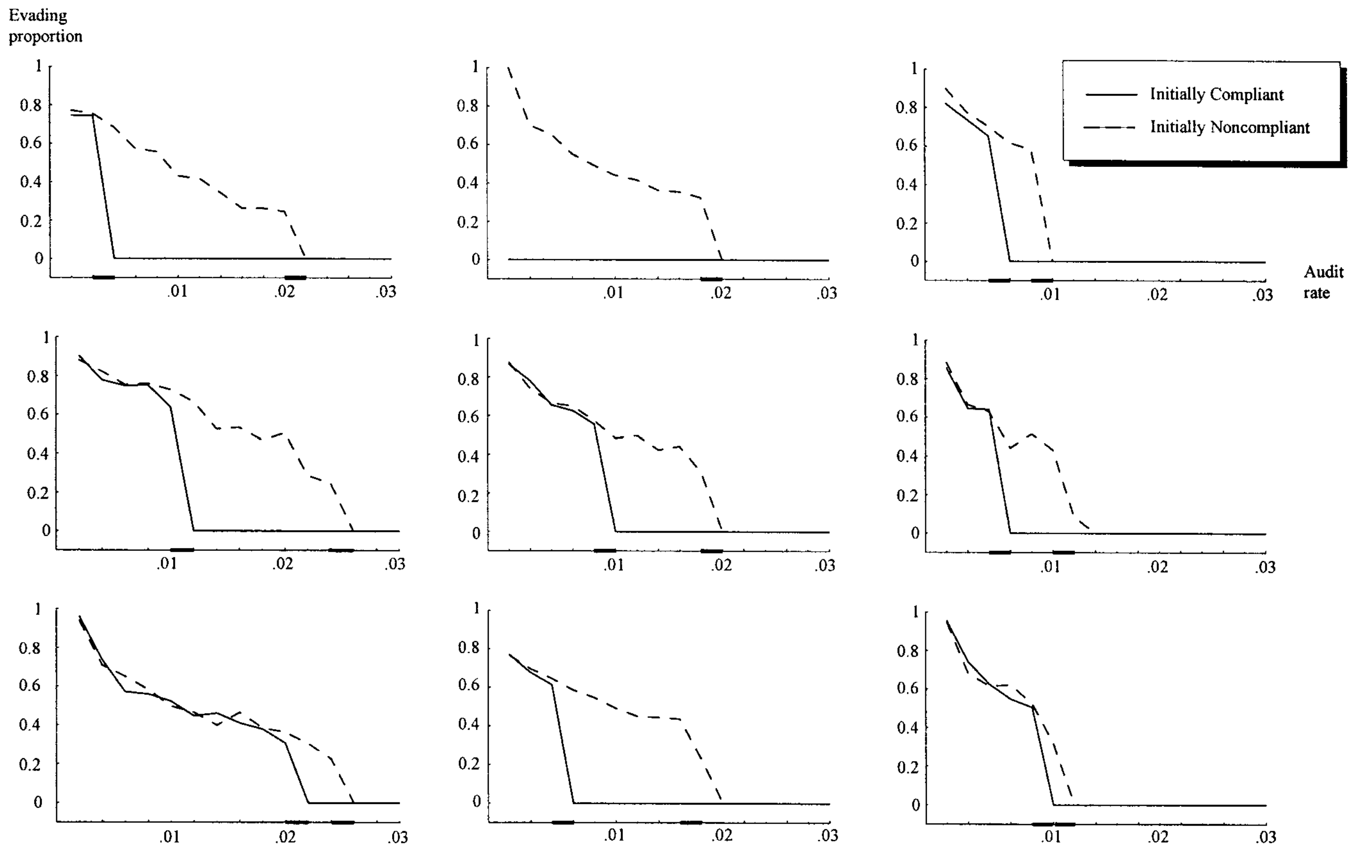
In this paper we examined the effects of others' behavior, enforcement, and social norms on the dynamics of taxpayer compliance. We employed two novel modeling methods. In our aggregate model, we found that two stable equilibria occur at some enforcement levels. The effect of changing enforcement levels on taxpayers' compliance depended on whether the initial population was relatively compliant or noncompliant: (1) in a population that is initially compliant, the tax agency can reduce enforcement down to a critical point with little effect on compliance, and (2) in a population that is initially noncompliant, increasing enforcement leads to modest increases in compliance up to a critical point. In the case of

[22] Because agent-based analysis is data driven, it can only falsify (and not verify) predictions. Although the data arising from our analysis are consistent with a Lyapunov-stable system, we cannot prove that our model is Lyapunov stable.

[23] The tax agency did not audit compliant taxpayers in this series of simulations.

[24] We investigated finer partitions (at 0.001 intervals) of audit rates around the sudden transition in behavior, and our results remain consistent with the aggregate model.

FIGURE 7
Evasion Across Audit Rates When Initial Populations Are Either Compliant or Noncompliant in the Agent-Based Model



Plot of the level of taxpayer evasion by audit rate after 2,000 time steps, for nine pairs of starting populations, where the members of each pair differ only in the level of compliance in the initial condition (10 percent evading vs. 50 percent evading). The darkened segments on the audit-rate axis indicate intervals where we observed sudden changes in equilibrium values.

a compliant equilibrium, however, our analysis indicated that when the tax agency decreases enforcement to just below some critical point, a dramatic shift in behavior will occur. The population will undergo a sudden and profound change, moving from compliance to noncompliance in equilibrium. Once this change occurs, attempts to remedy the sudden proliferation of evasion by increasing enforcement to the level in place before the change in behavior will be unsuccessful; the population will remain noncompliant. Similarly, if the tax agency increases enforcement above some critical point in an initially noncompliant population, the population will suddenly become compliant in equilibrium. Then, reducing enforcement to its previous level, below the critical point, will not lead to a significant increase in evasion. These results were robust; evaluation of more complex models with additional flows between taxpayer classes yielded equivalent results.

In contrast to our aggregate model, the agent-based model characterized individual decisions rather than aggregate behavior. Despite relaxing assumptions regarding movement between taxpayer classes and the effect of social norms, the agent-based model's results are generally consistent with the predictions of the aggregate model.

The models yield two major insights. First, in mostly honest populations, the tax agency might view enforcement as a mechanism to prevent evasion epidemics rather than as a way to improve existing compliance. Second, the possibility of sudden, dramatic changes in equilibrium behavior, together with the difficulty of reversing these changes through increased enforcement, should concern policymakers.

Our analysis suggests a number of avenues for future research. First, although tax compliance in the United States (and in some other countries) is arguably in a mostly honest state, our models predict that, at some point, a slight reduction in enforcement efforts may lead to a tax evasion epidemic. However, our work does not identify the level of enforcement that would trigger a sudden, dramatic shift to noncompliance. In the agent-based model, the shift occurred at different audit rates across different initial conditions, so the distribution of individual characteristics in a society may affect the point at which this transition occurs. Further investigation can determine what factors (if any) one might use to predict the transition point.

Continuing with our epidemic analogy, one also might evaluate the extent to which the taxpaying population can be "inoculated" against evasion (in particular, what psychological or sociological mechanisms might be brought to bear and how effective such mechanisms might be) and the aggregate consequences of inoculating a proportion of the population (e.g., how much of the population must be protected to eliminate the possibility of an evasion epidemic). While in a different context, King's (2002) work examining the extent to which group membership reduces self-serving biases in auditors serves as an analogy. From an analytical perspective, the herd-immunity literature might provide insights (e.g., see Edelstein-Keshet 1988).

Third, although we find similar results based on two different modeling approaches, an experimental investigation could lend even more credence to our findings. Borrowing from the experimental literature in social psychology and accounting that creates social groups and norms in the laboratory (e.g., Smith et al. 1998; Schmader and Major 1999; King 2002), one might establish an experimental environment like the one our model describes to test its predictions.

Researchers might use the class of models we employ to address other questions in accounting where the behavior of others influences one's own behavior. In auditing, researchers often characterize reputation as arising from one's personal experience with an auditor (Mayhew et al. 2001). Using our approach, one could expand the notion of reputation to explicitly recognize that one's notion of an auditor's reputation probably arises both from one's own experience and from the experiences and beliefs of others. We speculate that recognition of the influence of others could lead to some unexpected insights, such as robustness of reputation in the face of frequent audit failures. Similarly, norms established in the profession, by competitors' behaviors and by sanctions, likely affect behaviors that auditors view as acceptable from an independence perspective. Explicit recognition of these influences could lead to new insights regarding auditor behavior.

Researchers might also develop influence models to better understand financial reporting behavior (e.g., earnings management). According to Healy and Wahlen (1999), future research on earnings management should identify limiting factors. One might model the behavior of other firms and reporting norms as limiting factors. For example, if competitors engage in earnings management, then the firm may feel pressure to follow suit. Reporting norms may play a countervailing role. Our approach could also yield new insights into financial analyst behavior by explicitly accounting for the influence that analysts exert on one another.

In a broader sense, we believe that the methods we employ have the potential to contribute to accounting research by overcoming the tendency to draw from a single discipline, such as psychology, economics, or finance. Both of our modeling approaches are amenable to integration of ideas from across the social sciences. For example, we draw the first principles for our models from social psychology and sociology. In our analysis, we derive equilibria (viz., economics) and evaluate dynamics using techniques drawn from biology. Other work using dynamic systems analysis and agent-based models in the social sciences has integrated research from sociology, political science, and even anthropology to gain new understanding about societies, markets, and individual behavior.

APPENDIX
Local Stability Analysis

To evaluate the stability of the equilibria in our system, we consider populations that are slightly out of equilibrium. Assume that for any $x \in [0, 1]$, $g(x)$ has a convergent Taylor series near x and let $(\hat{x}_1, \hat{x}_2, \hat{x}_3)$ be an equilibrium population. Then, we can express $g(x_1)$ as:

$$g(x_1) = g(\hat{x}_1) + g'(\hat{x}_1)(x_1 - \hat{x}_1) + O[(x_1 - \hat{x}_1)^2] \tag{14}$$

for x_1 sufficiently near $\hat{x}_1$. We can then derive rates of change for the deviations from any equilibrium as follows:

$$\begin{aligned}
(x_1 - \hat{x}_1)' &= x_1' \\
&= [g(x_1) - rx_1]x_3 \\
&= (g(\hat{x}_1) - r\hat{x}_1 + [g'(\hat{x}_1) - r](x_1 - \hat{x}_1) + O[(x_1 - \hat{x}_1)^2])(x_3 - (\hat{x}_3 - \hat{x}_3)) \\
&= [g'(\hat{x}_1) - r]\hat{x}_3(x_1 - \hat{x}_1) + [g(\hat{x}_1) - r\hat{x}_1](x_3 - \hat{x}_3) \\
&\quad + [g'(\hat{x}_1) - r](x_1 - \hat{x}_1)(x_3 - \hat{x}_3) + O[(x_1 - \hat{x}_1)^2],
\end{aligned}$$

where $O[(x_1 - \hat{x}_1)^2]$ indicates that all remaining terms in the expression are second and higher powers of $(x_1 - \hat{x}_1)$. The last step is justified because $[g(\hat{x}_1) - r\hat{x}_1]\hat{x}_3 = 0$ if $(\hat{x}_1, \hat{x}_2, \hat{x}_3)$ is an equilibrium point. A similar analysis for $x_3 - \hat{x}_3$ yields:

$$
\begin{aligned}
(x_3 - \hat{x}_3)' &= x_3' \\
&= \alpha - \alpha x_1 - \alpha x_3 - g(x_1)x_3 \\
&= \alpha - \alpha(x_1 - \hat{x}_1 + \hat{x}_1) - \alpha(x_3 - \hat{x}_3 + \hat{x}_3) - [g(\hat{x}_1) + g'(\hat{x}_1)(x_1 - \hat{x}_1) \\
&\quad + O[(x_1 - \hat{x}_1)^2]](x_3 - \hat{x}_3 + \hat{x}_3) \\
&= \alpha - \alpha\hat{x}_1 - \alpha\hat{x}_3 - g(\hat{x}_1)\hat{x}_3 - \alpha(x_1 - \hat{x}_1) - [g(\hat{x}_1) + \alpha](x_3 - \hat{x}_3) \\
&\quad - g'(\hat{x}_1)(x_1 - \hat{x}_1)(x_3 - \hat{x}_3) + O[(x_1 - \hat{x}_1)^2] \\
&= -\alpha(x_1 - \hat{x}_1) - [g(\hat{x}_1) + \alpha](x_3 - \hat{x}_3) - g'(\hat{x}_1)(x_1 - \hat{x}_1)x_3 \\
&\quad + O[(x_1 - \hat{x}_1)^2].
\end{aligned}
$$

Again, the last step is justified because $\alpha - \alpha\hat{x}_1 - \alpha\hat{x}_3 - g(\hat{x}_1)\hat{x}_3 = 0$ at equilibrium. We combine the equations for $(x_1 - \hat{x}_1)'$ and $(x_3 - \hat{x}_3)'$ and write them in matrix form, leading to the following linearized system of equations near the equilibrium (we drop the higher-order term):

$$
\begin{aligned}
\begin{pmatrix} x_1 - \hat{x}_1 \\ x_3 - \hat{x}_3 \end{pmatrix}' &= \begin{pmatrix} [g'(\hat{x}_1) - r]\hat{x}_3 & g(\hat{x}_1) - r\hat{x}_1 \\ -\alpha & -[\alpha + g(\hat{x}_1)] \end{pmatrix} \begin{pmatrix} x_1 - \hat{x}_1 \\ x_3 - \hat{x}_3 \end{pmatrix} \\
&\quad + \begin{pmatrix} g'(\hat{x}_1) - r \\ -\, g'(\hat{x}_1) \end{pmatrix} (x_1 - \hat{x}_1)(x_3 - \hat{x}_3) + \begin{pmatrix} 0 \\ -g'(\hat{x}_1) \end{pmatrix} \hat{x}_3. \qquad (15)
\end{aligned}
$$

Now we examine the stability of the linearized system near the honest equilibrium. In particular, near a type (i) or honest equilibrium, the linearized system in Equation (15) takes the form:

$$
\begin{pmatrix} x_1 - 1 \\ x_3 \end{pmatrix}' = \begin{pmatrix} 0 & g(1) - r \\ -\alpha & -[\alpha + g(1)] \end{pmatrix} \begin{pmatrix} x_1 - 1 \\ x_3 \end{pmatrix} + \begin{pmatrix} g'(1) - r \\ -g'(1) \end{pmatrix} (x_1 - 1)x_3.
$$

The eigenvalues of the 2×2 matrix for the linear part of the system characterize the stability of the equilibrium at (1, 0, 0). Specifically, by the local stability theorem, (1, 0, 0) is stable if and only if both eigenvalues have negative real parts. It is unstable if either real part is positive. Hence, to prove Proposition 3, we must first find a λ to satisfy:

$$
\det \begin{vmatrix} -\lambda & g(1) - r \\ -\alpha & -\lambda - [\alpha + g(1)] \end{vmatrix} = 0.
$$

This gives the equation:

$$
\begin{aligned}
0 &= \lambda(\lambda + [\alpha + g(1)]) + \alpha[g(1) - r] \\
&= \lambda^2 + [\alpha + g(1)]\lambda + \alpha[g(1) - r].
\end{aligned}
$$

Solving the resulting quadratic equation for λ, we find:

$$\lambda = -\left(\frac{\alpha + g(1)}{2}\right)\left(1 \pm \sqrt{1 - \frac{4\alpha(g(1) - r)}{(\alpha + g(1))^2}}\right). \tag{16}$$

The roots are complex if and only if:

$$1 - \frac{4\alpha(g(1) - r)}{(\alpha + g(1))^2} < 0. \tag{17}$$

We rewrite this condition as:

$$[\alpha - g(1)]^2 < -4\alpha r. \tag{18}$$

Because α and r are positive, Equation (18) is never satisfied. The eigenvalues are real. The eigenvalue corresponding to $1 + \sqrt{\ldots}$ is negative. We determine stability using the sign of the other root. That is, (1, 0, 0) is unstable if and only if:

$$\begin{aligned} 1 - \sqrt{1 - \frac{4\alpha(g(1) - r)}{(\alpha + g(1))^2}} &< 0 \\ 1 &< \sqrt{1 - \frac{4\alpha(g(1) - r)}{(\alpha + g(1))^2}} \\ 0 &< -\frac{4\alpha(g(1) - r)}{(\alpha + g(1))^2} \\ 0 &< r - g(1) \\ g(1) &< r. \end{aligned}$$

Conversely, (1, 0, 0) is stable if and only if $g(1) > r$.

Extending our analysis to the linearized system near a type (ii) or mixed equilibrium, we obtain:

$$\begin{aligned} \begin{pmatrix} x_1 - \hat{x}_1 \\ x_3 - \hat{x}_3 \end{pmatrix}' &= \begin{pmatrix} [g'(\hat{x}_1) - r]\hat{x}_3 & 0 \\ -\alpha & -[\alpha + g(\hat{x}_1)] \end{pmatrix} \begin{pmatrix} x_1 - \hat{x}_1 \\ x_3 - \hat{x}_3 \end{pmatrix} \\ &+ \begin{pmatrix} g'(\hat{x}_1) - r \\ -g(\hat{x}_1) \end{pmatrix} (x_1 - \hat{x}_1)(x_3 - \hat{x}_3) + \begin{pmatrix} 0 \\ -g'(\hat{x}_1) \end{pmatrix} \hat{x}_3. \end{aligned} \tag{19}$$

The eigenvalues of the linear part are:

$$\lambda_1 = -[\alpha + g(\hat{x}_1)] \quad \text{and} \quad \lambda_2 = [g'(\hat{x}_1) - r]\hat{x}_3.$$

Because λ_1 is necessarily negative, we determine stability of the equilibrium by the sign of $g'(\hat{x}_1) - r$. This leads to Proposition 4.

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