The Effects of Integrity, Opportunity, Incentives, Mitigating Factors and Forensic Audit Procedures on Fraud Risk

By

Rajendra P. Srivastava, University of Kansas

Theodore J. Mock, University of Southern California

Jerry L. Turner, The University of Memphis

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ABSTRACT

This paper develops a comprehensive analytical model and closed-form formulas for fraud risk under the belief-function framework by incorporating the three components of the fraud triangle implied by SAS No. 99—Incentives/Pressures, Attitude/ Rationalizations, and Opportunities. We also include mitigating factors either internal to an entity, or imposed externally, such as those included in the Sarbanes-Oxley Act of 2002. The model and formulas facilitate explicit consideration of the interrelationships between fraud triangle components as well as the impact of forensic audit procedures that focus on specific fraud risks.

Our analyses show that fraud risk can be underestimated if interrelationships among the fraud triangle factors are not considered. As the strength of such interrelationships increase, both fraud risk and the possibility of understatement of that risk increases. Further analyses show that for clients with identified fraud risk factors, the presence of mitigating factors alone will not reduce fraud risk to an acceptably low level. In such cases, the auditors must perform forensic audit procedures to control fraud risk to an acceptable low level.

Key Words: Fraud Risk, SAS No. 99, Fraud Triangle, Sarbanes–Oxley

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I. INTRODUCTION

Two recent changes related to the accounting profession—promulgation of SAS No. 99 and enactment of the Sarbanes–Oxley Act of 2002 (the Act)—significantly affect both the risk of fraudulent financial reporting and the auditor's assessment of that risk. The objective of this paper is to develop an analytical model of fraud risk that incorporates the three components of the fraud triangle implied in SAS No. 99 and explicitly assesses the role of fraud-mitigating factors, including those contained in the Act, as well as the role of forensic audit procedures.

Development of a complete analytical model is important because only a general framework for fraud assessment exits presently. To fully understand the sources of fraud risk, interrelationships between those sources, and the impact of forensic audit procedures intended to reduce fraud risk, a formal model is required. Our model will allow researchers to evaluate and improve fraud risk assessment approaches and to evaluate the effectiveness of forensic audit procedures used to reduce fraud risk. Also, SAS No. 99 requires a pre-audit discussion by members of the audit team regarding the potential for material misstatement due to fraud (AICPA 1997, ¶14). Practitioners can use this model as a basis for that discussion. The model also can be used to educate less-experienced auditors about fraud risk, the need to identify interrelationships among the components of the fraud triangle and the need to select appropriate forensic audit procedures in response to the risk assessment.

Turner et al. (2002) argue that fraud risk and error risk should be integrated to obtain an overall audit risk and demonstrate how not incorporating fraud risk in the overall audit risk model understates the overall audit risk and thus may lead to an ineffective audit. However, they do not develop an analytical model in a closed form. The results of the present research would be of great value in that endeavor, i.e., in developing a closed form model of overall audit risk for planning and evaluation of audit and assurance services. While we understand that integrating fraud risk with the overall audit risk is important, because of space limitations we focus our efforts first on developing a fraud risk model. A future article will extend the present work by integrating this fraud risk model into an overall audit risk model.

We contribute to the current literature by extending the work of Dutta et al. (1998) who develop an audit risk model incorporating misstatement due to fraud, and that of Turner et al. (2002). The model developed by Dutta et al. (1998), lacks several important dimensions needed to evaluate fraud risk within the current professional environment. First, their treatment is limited only to affirmative items of evidence (see also Srivastava and Shafer 1992). More importantly, their model does not consider the impact of fraud triangle factors, does not incorporate any interrelationships among the fraud triangle factors, and does not consider the role of mitigating factors. As demonstrated in Section III, recognition of interrelationships among the fraud triangle factors is critical in estimating an overall risk of fraud and that fraud risk may be significantly underestimated if such interrelationships are not taken into account.

Mock and Turner (2001, 2003) argue that in post-SAS No. 82 audits, although auditors are required to document fraud risk factors identified on an engagement and to document corresponding audit program modifications, in some cases modifications may not reduce effectively the fraud risk identified on the engagement. For example, in one audit examined, the auditor simply extended planned audit procedures in response to identified fraud risk factors even though circumstances indicated that tests of an alternative nature might be required in order to control fraud risk. One can posit this could be due to the lack of a conceptual framework on how fraud risk should be integrated with the traditional audit risk. Turner et al. (2002) explore this situation but their analysis is limited in several ways. First, they do not consider the full impact of SAS No. 99 and the Act in their analysis. Second, they do not derive an analytical formula that incorporates both the additional fraud risk and mitigating factors described in SAS No. 99 and the Act and that also considers forensic audit procedures and interrelationships among the fraud triangle factors. To capture such additional influences, we develop a general analytical model that incorporates the SAS No. 99 fraud triangle.

In developing our model, we use the Dempster-Shafer theory of belief functions used by Srivastava and Shafer (1992) and many other researchers (see Srivastava and Mock 2000). In developing the model, we assume that readers have a basic familiarity with belief functions.[1]

Background

In 2002, the Auditing Standards Board released SAS No. 99, *Consideration of Fraud in a Financial Statement Audit* (AICPA 2002), intended to expand required audit procedures assessing the risk of material financial statement fraud.[2] The SAS emphasizes considering a

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[1] For basics and applications of belief functions in accounting and auditing, see Srivastava and Mock (2000), Srivastava (1993), Srivastava and Shafer (1992), and Shafer and Srivastava (1990).

[2] Although the AICPA Auditing Standards Board no longer has authority to set audit standards for publicly traded companies, SAS No. 99 still remains in effect. SAS No. 99 identifies two

client's susceptibility to fraud, regardless of the auditor's past experience with the entity or prior beliefs about management's honesty and integrity. The definition of audit risk, i.e., the risk that the auditor may unknowingly fail to appropriately modify his opinion on financial statements that are materially misstated (AICPA 1983, ¶02), was not modified by SAS No. 99. Rather, SAS No. 47 (AICPA 1983, ¶02) previously indicated that the risk of material misstatement due to fraud is part of audit risk. SAS No. 99 failed, however, to clearly describe where fraud risk is incorporated in the components of audit risk—inherent risk, control risk and detection risk. While this is an important issue, for purposes of our analyses, how fraud risk integrates into the full audit risk model is left to future research.

Ramos (2003) summarizes the requirements of SAS No. 99 and discusses the concept of "The Fraud Triangle," consisting of three conditions generally present when fraud occurs: Incentives/Pressures, Attitude/ Rationalizations, and Opportunities.[3] Forensic experts, academics and others argue that evaluation of information about fraud is enhanced when auditors consider it in the context of these three conditions (Montgomery et al. 2002). Although SAS No. 99 never refers to the term "Fraud Triangle", the three conditions are identified and discussed and the term has become popularly accepted in referring to SAS No. 99.

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categories of fraud—fraudulent financial reporting and misappropriation of assets (¶6). This paper focuses on the former, but the analytical approach for the latter is identical in nature.

[3] For brevity, the remainder of this paper refers to incentives/pressures only as *incentives* unless specifically discussing circumstances involving pressures, and attitudes/rationalizations are referred to as *integrity.*

SAS No. 99 emphasizes obtaining a broader range of information to serve as the foundation for an assessment that goes beyond considering the fraud risk factors provided in SAS No. 82 and beyond the guidance and requirements included in international auditing standards.[4] The various sources of information—audit team discussions, inquiries of management and others, consideration of fraud risk factors, results of planning analytical procedures, information from the client acceptance or continuance process and from reviews of interim financial statements—are compounded into the auditor's evaluation of fraud risk. However, SAS No. 99 provides little guidance as to how factors affecting the evaluation are to be combined into a quantifiable measure.

In discussing issues that might influence the evaluation process, SAS No. 99 (¶20) describes mitigating factors as "Programs and controls the entity has established to mitigate specific fraud risks the entity has identified, or that otherwise help to prevent, deter, and detect fraud, and how management monitors those programs and controls." Such programs and controls may involve "(a) specific controls designed to mitigate specific risks of fraud—for example, controls to address specific assets susceptible to misappropriation, and (b) broader programs designed to promote a culture of honesty and ethical behavior" (¶44). While specific programs and controls may reduce the risk of fraud, SAS No. 99 (¶44) also warns that specific control deficiencies actually may exacerbate the risk of fraud.

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[4] In regard to the nature of guidance provided and audit procedures required, current international auditing standards are related more closely to SAS No. 82 than to its successor, SAS No. 99.

SAS No. 99 notes "The auditor's response to the assessment of the risks of material misstatement of the financial statements due to fraud is influenced by the nature and significance of the risks identified as being present and the entity's programs and controls that address these identified risks" (¶47). Paragraph 48 of the SAS identifies three ways the auditor may respond to identified fraud risks, two of which are considered in our analyses[5]:

- A response involving the nature, timing and extent of auditing procedures to be performed, and
- A response addressing the risk of fraud due to management override of controls.

For our analysis, we combine the first two types of response, but consider only those procedures modified or added specifically to address the risk of fraud identified during the fraud risk assessment. We define these fraud-specific procedures as forensic audit procedures. While audit procedures intended to identify errors related to a financial statement assertion also may

[5] The third suggested way is, "A response that has an overall effect on how the audit is conducted, such as the assignment of personnel and supervision, exhibiting greater concern over the selection of accounting principles, or incorporating an element of unpredictability in the year-to-year selection of audit procedures to be performed." Integration of this type of response into our model is problematic as no direct evidence related to a specific assertion is generated. However, one could incorporate the effects of such procedures through the beliefs obtained for the affected assertions.

provide some level of belief regarding the absence of fraud, the design of such procedures does not normally focus on the risk of fraud.[6]

The second significant change that may affect the assessed level of fraud risk is promulgation of the Act. The Act codifies significant changes to corporate governance that mitigate incentives and opportunities, and encourages greater integrity on the part of management. For example, Section 301 of the Act strengthens the role of the audit committee of the board of directors, and requires establishing procedures for the receipt, retention, and treatment of complaints received by the issuer regarding accounting, internal controls, and auditing. Section 302 requires the CEO and CFO of each issuer to prepare a statement to accompany the audit report to certify the "appropriateness of the financial statements and disclosures contained in the periodic report, and that those financial statements and disclosures fairly present, in all material respects, the operations and financial condition of the issuer." Section 303 makes it unlawful for any officer or director of an issuer to take any action to fraudulently influence, coerce, manipulate, or mislead any auditor engaged in the performance of an audit for the purpose of rendering the financial statements materially misleading.

Other sections of the Act also are intended to reduce the risk of fraud. Section 402(a) prohibits loans to any director or executive officer, thereby reducing both incentives and opportunities. Section 404 requires each annual report of an issuer to contain an "internal control

[6] This type of complexity can be modeled and analyzed (see Turner et al. 2002). However, it renders the evidential model to be a network and complicates the analyses significantly. Moreover, we are interested in developing analytical formulas for audit risk under various scenarios. This is not possible if the evidential diagram becomes a network.

report", which (1) states the responsibility of management for establishing and maintaining an adequate internal control structure and procedures for financial reporting; and (2) contains an assessment, as of the end of the issuer's fiscal year, of the effectiveness of the internal control structure and procedures of the issuer for financial reporting. RQ2 and RQ3 address the effects of both specific and general mitigating factors on fraud risk.

Research Questions

SAS No. 99 implies, but does not formalize, interrelationships between the three components of the fraud triangle:

> …even otherwise honest individuals can commit fraud in an environment that imposes sufficient pressure on them. The greater the incentive or pressure, the more likely an individual will be able to rationalize the acceptability of committing fraud (¶7).

Using the general analytical model developed in Appendices A and B, we first investigate the impact of interrelationships among the fraud triangle factors:

RQ1: What is the impact on fraud risk of interrelationships among fraud triangle factors?

After examining the impact of interrelationships, we discuss several special cases and examine three additional research questions investigating the impact of the presence or absence of management integrity, incentives, opportunity, mitigating factors, and forensic audit procedures on fraud risk. Our remaining research questions are:

RQ2: What is the impact on fraud risk of the presence or absence of individual components of the fraud triangle and of the implementation of mitigating factors such as those introduced by the Sarbanes-Oxley Act of 2002?

RQ3: What is the combined impact on fraud risk of the presence or absence of multiple components of the fraud triangle?

RQ4: What is the impact on fraud risk of performing forensic audit procedures?

The remainder of this paper is organized as follows. Section II first develops the mathematical equations for the basic components of the fraud triangle and then adds forensic audit procedures and interrelationships. In Section III, we examine the sensitivity of the model to various levels of evidence and how variation in the components of the fraud triangle, individually and in combination, affects fraud risk. In particular, we analyze the effects on fraud risk of risk factors and mitigating factors implemented by management or required by the Act. In Section IV, we summarize our results. Finally, in Appendix A we present a step-by-step discussion of the derivation of the general equations for fraud risk and belief in fraud used in Section II and in Appendix B, we extend the general equations to include the effects of mitigating factors and forensic audit procedures.

II. FORMAL FRAUD RISK FORMULAS

In developing a comprehensive model of fraud risk, we begin by examining a simple model incorporating only the three components of the fraud triangle. The model then is enhanced by adding forensic audit procedures and by considering mitigating factors. Finally, the model is completed by adding interrelationships between the fraud triangle components.

Basic Fraud Risk Model with Unrelated Fraud Triangle Factors, with Mitigating Factors, but No Forensic Audit Procedures

In Appendix A, we use a belief-function framework to combine items of evidence pertaining to the three fraud risk factors: incentives, integrity, and opportunities. Figure 1 presents an evidential diagram for fraud risk that relates the three fraud risk categories to the assertion "No Fraud" with an 'OR' relationship. In general, there are two sources of possible evidence for each category. The first source of evidence relates to the impact of identified fraud risk factors about the belief that fraud may exist. SAS No. 99 provides an appendix containing an extensive, but not complete, list of fraud risk factors an auditor might identify during the assessment process. The second source of evidence relates both to the impact of specific mitigating factors related to identified fraud risk factors and to more general mitigating factors, such as those included in the Act.

Insert Figure 1 about here

When evaluating the entity and its environment under SAS No. 99, the auditor must consider whether the information indicates one or more fraud risk factors are present. Identification of fraud risk factors involves the application of professional judgment and includes consideration of the attributes of the risk, including the *type* of risk that may exist, the *significance* of the risk, the *likelihood* of the risk, and the *pervasiveness* of the risk (AICPA 2002, ¶40). SAS No. 99, Appendix A, provides examples of fraud risk factors, categorized as factors related to incentives/pressures to perpetuate fraud, to an attitude/rationalization to justify the fraudulent action, and to the opportunity to carry out the fraud.

Srivastava and Shafer (1992) have argued that under the belief-function framework, the plausibility of errors, Pl(errors), represents audit risk[7]. Similarly, we define fraud risk to be the plausibility of fraud, Pl(Fraud), and derive a general expression of fraud risk [see equation (A7) and equation (A8) in Appendix A] in terms of the product of three plausibilities or risks—incentives exist, management lacks integrity, and there are opportunities:

$$\text{Pl(Fraud)} = \text{Fraud Risk} = \text{Pl(Incentives)} \times \text{Pl(Lack of Integrity)} \times \text{Pl(Opportunities)} \qquad (1)$$

Note that the general equation in (A8) contains the interaction terms, r_1 and r_2, through the renormalization constant K. These interaction terms represent the impact of interrelationships among the fraud risk factors. However, in the present case, we assume there are no relationships among the three fraud risk factors, i.e., $r_1 = r_2 = 0$, which yields K =1 in A7. As a result, we obtain the fraud risk formula given in equation (1) above. However, since there are risk factors and mitigating factors pertaining to each fraud triangle component as discussed earlier, we need to incorporate the effects of theses factors on fraud risk. The first part of Appendix B deals with this issue. The fraud risk formula with individual risks is (see B4 in Appendix B for details):

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[7] There are three important functions in the belief-function framework that are relevant to the present discussion. First is the *basic probability assignment function* or *m-values* (Shafer 1976). Suppose there are n possible values of variable A, say $a_1, a_2, a_3 \ldots a_n$. The set of values is called a frame and is denoted by $\Theta = \{a_1, a_2, a_3 \ldots a_n\}$. In general, one can assign m-values (beliefs) to each single element of the frame, all subsets of two elements, three elements, and so on to the entire frame such that sum of all these m-values is 1, i.e., $\sum_{A \subseteq \Theta} m(A) = 1$. Second is the belief on a subset of elements, say A, of the frame, which is defined as: $\text{Bel}(A) = \sum_{B \subseteq A} m(B)$. Bel(A) represents the total belief that A is true based on the m-values for A and the subsets contained in A. Third is the plausibility of a subset, say A, of the frame, which is defined as Pl(A) = 1 - Bel(~A). That is, the plausibility of A is one minus the belief that it is not A. For a detailed discussion on belief functions see Srivastava and Mock (2002).

$$\text{Fraud Risk} = \text{Pl(Fraud)} = \frac{(R_{RF}^{NI})(R_{MF}^{NI})}{K_{NI}} \times \frac{(R_{RF}^{IT})(R_{MF}^{IT})}{K_{IT}} \times \frac{(R_{RF}^{NO})(R_{MF}^{NO})}{K_{NO}} \quad (2)$$

where R_{RF}^{NI}, R_{MF}^{NI}, R_{RF}^{IT}, R_{MF}^{IT}, R_{RF}^{NO}, and R_{MF}^{NO} are defined in Table 1.

The factors in the denominator in equation (2) represent renormalization constants originating from the use of Dempster's rule for combining two items of evidence (Shafer 1976), one from the risk factors and the other from the mitigating factors, pertaining to a single variable that may have conflicts. These factors are defined as (see Table 1 for symbol definitions):

$$K_{NI} = 1 - m_{RF}(ni)m_{MF}(\sim ni) - m_{RF}(\sim ni)m_{MF}(ni),$$

$$K_{IT} = 1 - m_{RF}(it)m_{MF}(\sim it) - m_{RF}(\sim it)m_{MF}(it),$$

$$K_{NO} = 1 - m_{RF}(no)m_{MF}(\sim no) - m_{RF}(\sim no)m_{MF}(no),$$

where m..(..) represent the basic beliefs obtained from the corresponding risk factors and mitigating factors.

Insert Table 1 about here

Also, from (A4-A6) in Appendix A, we can write the belief in fraud, Bel(fraud), and belief in 'no fraud', Bel(no fraud), for the condition with no relationships among the three factors (i.e., $r_1 = r_2 = 0$) as:

$$\text{Bel(no fraud)} = 1 - \text{Pl(Incentives)} \times \text{Pl(Lack of Integrity)} \times \text{Pl(Opportunities)} \quad (3)$$

$$\text{Bel(fraud)} = \text{Bel(Incentives)} \times \text{Bel(Lack of Integrity)} \times \text{Bel(Opportunities)} \quad (4)$$

Note that plausibilities and beliefs on the right side of equations (2-4) are the combined plausibilities and beliefs of the two items of evidence—fraud risk factors and mitigating factors—for each fraud risk component shown in Figure 1.

Analysis of the Model

As can be seen from equations (A1-A4), in the absence of any evidence about the presence of the three fraud risk factors, 'No Incentives (NI)', 'Integrity (IT)', and 'No Opportunity (NO)', (i.e., $m_{NI}(ni) = m_{NI}(\sim ni) = 0$, $m_{NI}(\{ni, \sim ni\}) = 1$; $m_{IT}(it) = m_{IT}(\sim it) = 0$, $m_{IT}(\{it,\sim it\}) = 1$; and $m_{NO}(no) = m_{NO}(\sim no) = 0$, $m_{NO}(\{no,\sim no\}) = 1$), we obtain the following values for fraud risk and the beliefs that there is fraud and that there is no fraud:

Fraud Risk = $Pl(\sim nf) = 1$,

$Bel(fraud) = 0$, and

$Bel(nf) = 0$.

These results are logical because if there is no evidence of any kind, the belief that there is fraud or there is no fraud should be zero, whereas the plausibility [risk] that there is fraud or there is no fraud should be 1. Equation (2) suggests that the fraud risk may be reduced to an acceptably-low level by reducing the risk in any one, two, or all three categories of fraud risk factors. The risks related to these factors can be reduced either by identifying strong mitigating factors or by performing appropriate forensic audit procedures such as those required by SAS No. 99.

Basic Fraud Risk Model with Forensic Procedures

We now extend equation (2) to include the beliefs from forensic audit procedures performed on the associated accounts. While the modified expression for the overall fraud risk that there is fraud is derived in Appendix B (see B10) in its full generality with the presence of interrelationships among fraud triangle factors, in the present discussion we assume that the three fraud triangle factors are not related. As mentioned earlier, under this condition, $r_1 = r_2 = 0$, which reduces K, the renormalization constant, in (B10) to 1. Thus, the modified fraud risk formula for this case is obtained from (B10) by setting K =1:

$$\text{Fraud Risk} = \text{Pl(Fraud)} = \frac{\left[\dfrac{(R_{RF}^{NI})(R_{MF}^{NI})}{K_{NI}} \text{ x } \dfrac{(R_{RF}^{IT})(R_{MF}^{IT})}{K_{IT}} \text{ x } \dfrac{(R_{RF}^{NO})(R_{MF}^{NO})}{K_{NO}}\right] \text{x} R_{FP}}{K_0} \qquad (5)$$

where R_{FP} = Risk that forensic procedures fail to detect fraud (see also Table 1).

K_0 = Renormalization constant for conflicting evidence as defined in (B8)

and other factors are as defined previously.

The belief that there is no fraud based on all the evidence gathered can be written as:

$$\text{Bel(no fraud)} = 1 - \text{Fraud Risk} \qquad (6)$$

Similar to equation (2), equation (5) suggests that the fraud risk can be reduced to an acceptable level by reducing the risk in any combination of one or more of the four factors. That is, one way to achieve an acceptable low level of fraud risk is by reducing the risk related to incentives, lack of integrity, and/or opportunities, or through strong mitigating factors. However, if no strong mitigating factors can be identified, the only way fraud risk can be reduced to an acceptably-low level is through evidence obtained by performing forensic audit procedures. Equation (5) not only is appropriate for planning the audit for a desired level of fraud risk but also for evaluation purposes as it is an exact formula with included evidence being positive, negative or mixed.

Fraud Risk Model with Forensic Procedures and Interrelationships between Fraud Triangle Factors

In Figure 1, the three categories of fraud triangle factors are connected by two relationships—one between 'No Incentives' and 'Integrity', designated as r_1 and a second between 'Integrity' and 'No Opportunities', designated as r_2. As discussed in Appendix A, each relationship is bidirectional in its influence. For example, in the case of r_1, if there is belief that management has incentives to commit fraud then this relationship implies that there is a non-zero belief that management lacks integrity. Similarly, if there is evidence that management lacks integrity then this relationship implies that there is non-zero belief that management might create incentives to commit fraud. This relationship also implies that if management has integrity then the effect of management incentives to commit fraud is negated or if there are no incentives then management would behave with integrity (see Appendix A for a detailed discussion).

No relationship is shown between Incentives and Opportunities because it is less obvious that these two components directly affect one another. For example, existence of an incentive does not create an opportunity to obtain that incentive. Also, existence of an opportunity, such as a control deficiency, does not necessarily create a corresponding incentive for fraud resulting from that deficiency. However, the third relationship can be implied by providing non-zero values for both the r_1 and r_2 relationships.

These relationships can be modeled in several ways. For example, one can assume that each takes a fixed value, say r_1 and r_2, thus establishing the level of interaction, or one can assume that the value of each depends on the level of beliefs about the related variables. Under either assumption, $0 \leq r_1 \leq 1$, and $0 \leq r_2 \leq 1$.

If we assume the relationships to be dependent on the level of belief about the variables involved, then the impact of the relationship, which represents a two-way interaction, would vary with the degree of belief in the variables. For example, under this assumption, the r_1 relationship recognizes that at lower levels of incentives, a manager may exhibit very high integrity and will not respond to such incentives. At higher levels of incentives, however, that same manager may decide that the incentives may be so compelling that the manager's integrity is compromised and the rewards offered are sought actively. The relationship also recognizes that a manager with a lower level of integrity may seek to create or increase existing Incentives.

Similarly, under the above assumption, the r_2 relationship creating a possible two-way interaction between Integrity and Opportunities recognizes that at lower levels of opportunities, a manager may exhibit very high integrity and will not respond to afforded opportunities. If greater opportunities are presented, however, that same manager may decide that the opportunities may be so available that the manager's integrity is compromised and fraud risk is increased. The resulting compromise in Integrity may result in the manager identifying or creating incentives that might become available. The relationship also recognizes that a manager with a lower level of integrity may seek to create or increase existing opportunities.

Equations (A5-A7) in Appendix A, respectively, represent the overall belief that there is no fraud, the overall belief that there is fraud, and the overall plausibility that there is fraud based on the three fraud risk factors with the added assumption that Incentives and Integrity are interrelated, and that Integrity and Opportunities are interrelated. To derive the fraud risk model with forensic procedures along with fraud triangle factors and their relationships, we use Dempster's rule to combine the beliefs in 'fraud' and 'no fraud' as defined in (A5) and (A6) with the beliefs in fraud and no fraud obtained from forensic procedures. This process is described in the second part of Appendix B and yields the following expressions for the overall plausibility in fraud and overall beliefs in fraud and no fraud (see B5-B9):

$$Pl_T(\sim nf) = \text{Fraud Risk} = Pl(\sim ni)Pl(\sim it)Pl(\sim no)Pl_{FP}(\sim nf)/KK_0, \quad (7)$$

$$Bel(nf) = 1 - Pl(\sim ni)Pl(\sim it)Pl(\sim no)Pl_{FP}(\sim nf)/KK_0, \quad (8)$$

$$Bel(\sim nf) = [m_H(\sim nf)m_{FP}(\sim nf) + m_H(\sim nf)m_{FP}(\Theta_{NF}) + m_H(\Theta_{NF})m_{FP}(\sim nf)]/KK_0. \quad (9)$$

where the values of m_H are defined in Equations (A1-A3), and K and K_0 are defined as follows (see also Equation A4 in Appendix A for K, and Appendix B for K_0):

$$K = 1 - m_{IT}^{+}(r_1 m_{NI}^{-} + r_2 m_{NO}^{-} - r_1 r_2 m_{NI}^{-} m_{NO}^{-}) - m_{IT}^{-}(r_1 m_{NI}^{+} + r_2 m_{NO}^{+} - r_1 r_2 m_{NI}^{+} m_{NO}^{+})$$
$$- r_1 r_2 m_{IT}^{\Theta}(m_{NI}^{+} m_{NO}^{-} + m_{NI}^{-} m_{NO}^{+}), \quad (10)$$

$$K_0 = 1 - [m_H(\sim nf)m_{FP}(nf) + m_H(nf)m_{FP}(\sim nf)]. \quad (11)$$

Again using the definition of risk in terms of plausibility as defined by Srivastava and Shafer (1992), we obtain the following fraud risk model for the full complex model that includes the three fraud risk categories along with their interrelationships and the impact of mitigating factors and forensic audit procedures (see B10 in Appendix B).

$$\text{Fraud Risk} = Pl(\text{Fraud}) = \frac{\left[\frac{(R_{RF}^{NI})(R_{MF}^{NI})}{K_{NI}} \text{x} \frac{(R_{RF}^{IT})(R_{MF}^{IT})}{K_{IT}} \text{x} \frac{(R_{RF}^{NO})(R_{MF}^{NO})}{K_{NO}}\right] \text{x} R_{FP}}{KK_0} \quad (12)$$

where K = a renormalization factor for conflicting evidence resulting from interrelationships as defined in equation (10) and other factors are as defined previously. We emphasize that the fraud risk formula derived in (12) is the most general formula that incorporates the effects of SAS No. 99 risk factors, mitigating factors, the effects of forensic audit procedures, and the interrelationships among the fraud triangle factors. It also includes positive, negative and mixed items of evidence. This model can be used both for planning and for evaluation of fraud risk on any engagement.

The impact of interrelationships on the overall fraud risk as given in (12) is through the factor K (defined in Equation 10) in the denominator. K is 1 when r_1 and r_2 are assumed to be zero, that is, when the interrelationships between incentives and integrity and between integrity and opportunities are assumed to be non-existent. As can be seen from (10), K decreases as r_1 and r_2 increases, resulting in an increase in the fraud risk. In other words, in general, fraud risk will increase with an increase in the strength of interrelationships among fraud triangle risk components.

III. SENSITIVITY ANALYSIS OF THE FRAUD RISK MODEL

To analyze the sensitivity of the fraud risk model to various types and levels of evidence, we assign values to the evidential nodes, individually and in combination, and then calculate the effects of such evidence on fraud risk. To accomplish this, we program the general model in equation (12) in a spreadsheet and use special cases to analyze the results.

The first research question, RQ1, deals with determining the impact on fraud risk of interrelationships among the fraud triangle factors.

RQ1: What is the impact on fraud risk of interrelationships among fraud triangle factors?

Figure 2 presents this impact for the following assumed situation:

(1) we have no evidence about the management integrity either from risk factors (RF) or from mitigating factors (MF), i.e., $m_{RF}(it) = m_{RF}(\sim it) = 0$, and $m_{MF}(it) = m_{MF}(\sim it) = 0$,

(2) we have the following beliefs for management incentives from the two sources: $m_{RF}(ni) = 0$, $m_{RF}(\sim ni) = 0.8$; $m_{MF}(ni) = 0.5$ $m_{MF}(\sim ni) = 0$, and

(3) we have the following beliefs for opportunities from risk factors and mitigating factors relevant to opportunities: $m_{RF}(no) = 0$; $m_{RF}(\sim no) = 0.8$; $m_{MF}(no) = 0.5$, $m_{MF}(\sim no) = 0$.

(4) The belief value for evidence resulting from forensic procedures, $m_{FP}(nf)$, varies from 0 to 1.

These assumptions are summarized in Table 2.

Insert Table 2 about here

The latter set of m-values suggest that risk factors are assumed to support the existence of opportunities at a 0.8 level of belief on a scale of 0 to 1, and the corresponding mitigating factors provide evidence that opportunities are not readily available (i.e., the evidence supports the non-existence of opportunities) at a 0.5 level of belief. Similar interpretations can be extended for the beliefs about incentives.

Figure 2 graphs fraud risk as a function of the belief in support of ‘no fraud’ obtained from forensic procedures (FP). As evidenced from the graph, fraud risk increases with the increase in the strength of relationships, r_1 and r_2. This increase is non-linear as one can see from the graphs that the increase in fraud risk from $r_1 = r_2 = 0.5$ to $r_1 = r_2 = 1.0$ is significantly higher than the increase from $r_1 = r_2 = 0$ to $r_1 = r_2 = 0.5$.

Insert Figure 2 About Here

These findings are important as they show that without considering the interrelationships among the fraud factors, the auditor potentially will underestimate fraud risk. This potential understatement increases at an increasing rate as r_1 or r_2 increases. Both effects may result in an ineffective audit.

For the remaining research questions, RQ2—RQ4, we use the assumed strengths of belief about evidence as shown in Table 2. In all these cases, we assume that the associated interrelationships vary with the associated level of belief.

RQ2: What is the impact on fraud risk of the presence or absence of individual components of the fraud triangle and of the implementation of mitigating factors such as those introduced by the Sarbanes-Oxley Act of 2002.

Direct evidence about Integrity only

To examine RQ2, we first focus on the middle component of the fraud triangle, Integrity, by assuming the auditor finds no direct evidence suggesting that fraud may exist except for one or more factors related to a possible lack of management integrity. As prescribed by SAS No. 99, the exercise of professional skepticism is important when considering the risk of material misstatement due to fraud and the auditor should be wary of overestimating management integrity:

> The auditor should conduct the engagement with a mindset that recognizes the possibility that a material misstatement due to fraud could be present, regardless of any past experience with the entity and regardless of the auditor's belief about management's honesty and integrity (AICPA 2002, ¶13).

In practice, there may be instances where management actions cause the auditor not only to question management's integrity, but also to believe that incentives and opportunities related to those actions exist, i.e., the belief that there are incentives and the belief that there are opportunities are greater than zero.

For example, one fraud risk factor listed in SAS No. 99 under Attitudes/Rationalizations is "Recurring attempts by management to justify marginal or inappropriate accounting on the basis of materiality" (AICPA 2002, Appendix). If this condition is observed, a logical conclusion is that management has some incentive to make these attempts and that management has the ability or opportunity to create the questionable accounting procedures. Therefore, identification of evidence about possible lack of integrity leads to a related belief in the existence of evidence about incentives and opportunities.

In establishing belief values for the r_1 and r_2 relationships, we assume each relationship varies with the strength of beliefs in the two related components of the fraud triangle. For r_1, for example, the more suspect is management's integrity, the stronger the belief that incentives might exist or be created. Also, the greater the incentive, the more suspect is management's integrity as it is more likely that a large incentive will sway the level of management's integrity than will a small incentive. For r_2, the more suspect is management's integrity, the stronger the beliefs that opportunities might exist or be created. Similarly, weak controls or the possibility of management override of existing controls may sway management's integrity in regard to taking advantage of those opportunities. To simplify this relationship, we define the values for r_1 and r_2

to be the average of the belief values for the related components, $r_1 = [m_{RF}(\sim ni) + m_{RF}(\sim it)]/2$ and $r_2 = [m_{RF}(\sim it) + m_{RF}(\sim no)]/2$. Inclusion of these relationships then completes the fraud triangle.

Figure 3 shows that as the auditor's belief about a lack of management integrity increases, mitigating factors become less effective at reducing fraud risk. In addition, regardless of the level of belief about a lack of management integrity, the reliance on mitigating factors must be extremely high to reduce fraud risk to a low level.

Insert Figure 3 about here

Evidence about either Incentives or about Opportunity only

Next, we examine the influence of evidence about the risk of fraud pertaining to one of the outer nodes of the model. In performing a fraud assessment, the auditor may identify evidence that certain incentives (opportunities) may exist, but find no direct evidence of a lack of management integrity or of opportunities (incentives). For example, SAS No. 99 identifies the following risk factor for incentives/pressures:

> There is excessive pressure on management or operating personnel to meet financial targets set up by the board of directors or management, including sales or profitability incentive goals (AICPA 2002, Appendix).

The auditor must view the presence of such pressure with professional skepticism, leading to a belief that management may be swayed to compromise its integrity to ease that pressure. Thus, the r_1 relationship allows a transfer of beliefs from the presence of incentives $m_{RF}(\sim ni)$ to a belief that management may respond inappropriately to those incentives $m_{RF}(\sim it)$. Once the auditor transfers a belief to the possible lack of management integrity, the auditor must assume that management is willing to and can find a method of achieving the incentive. The r_2 relationship then allows a further transfer of beliefs to opportunities m_{RF}(~no). For RQ3, we establish values for r_1 and r_2 in the same manner as RQ2.

Figure 4 shows that when evidence exists only at one of the outer nodes of the fraud triangle (Incentives or Opportunities), the results are identical in regard to fraud risk as when the evidence relates only to the center node (Integrity). This result provides an important insight into the fraud triangle in that when risk factors are identified in any one of the three nodes, the impact on fraud risk can be substantial and is functionally identical. Therefore, one category of fraud risk should not be weighted more heavily than either of the other two categories.

Insert Figure 4 about here

Figure 4 also shows the impact of mitigating factors on fraud risk. As seen from the graph, as the belief in 'no incentive' increases due to stronger mitigating factors, fraud risk decreases. However, to achieve a low level of fraud risk, one needs to have a very strong belief that mitigating factors actually serve to deter actions associated with the risk.

Our third research question examines the impact on fraud risk when evidence about the risk of fraud exists about any two of the three components of the fraud triangle.

RQ3: What is the combined impact on fraud risk of the presence or absence of multiple components of the fraud triangle?

Evidence about Integrity in combination with evidence about either Incentives or Opportunity

We first examine the impact of evidence at either of the outer nodes of the fraud triangle (Incentives or Opportunities) in combination with evidence about the center node indicating a possible lack of integrity. While no evidence about the third node is evident, the associated r_1 or r_2 relationship allows a propagation of belief from the Integrity node, again completing the fraud triangle. This implies that either (1) given strong-enough incentives and a compromised level of integrity, the auditor must believe that opportunities to acquire those incentives may be found or created, or (2) given opportunities and a compromised level of integrity, the auditor must believe that incentives to benefit from those opportunities may be found or created. For RQ3, we establish values for r_1 and r_2 in the same manner as RQ2.

As seen in Figure 5, when belief about evidence from either outer node (Incentives or Opportunities) is combined with a belief about a lack of integrity, the effect on fraud risk becomes more complex than when evidence relates only to a single node.

Insert Figure 5 about here

Evidence about Incentives in combination with evidence about Opportunity, but no evidence about Integrity

In an actual audit setting, evidence regarding lack of integrity, $m_{RF}(\sim it)$, or integrity, $m_{RF}(it)$, may be difficult to identify and evaluate, but evidence regarding the existence of incentives $m_{RF}(\sim ni)$ and opportunities $m_{RF}(\sim no)$ typically can be identified more easily. When r_1 and r_2 are defined as in RQ2, the result is a belief that a possible lack of integrity exists that in turn results in a non-zero belief in the possibility of fraud. This belief about the possibility of fraud reflects that given strong-enough incentives and available opportunities, even otherwise honest managers may be tempted to act inappropriately.

When evidence exists at both the Incentives and Opportunities nodes of the fraud triangle, the impact of r_1 and r_2 result in a belief that management may seek to take advantage of circumstances. Given this combination of evidence, the overall effect on fraud risk is somewhat less extreme than when an equivalent level of direct evidence about management integrity is available due to the diluting effect of the interrelationships as beliefs are propagated from the outer two nodes to the center node.

As seen in Figure 6, fraud risk increases as beliefs increase that incentives and opportunities exist. However, fraud risk can be decreased by identifying mitigating factors. The decrease is faster at a low level of beliefs in 'no incentives' and 'no opportunities' resulting from mitigating factors and is due mainly to the dependence of the interrelationships, r_1 and r_2, on the beliefs of the variables involved. As the beliefs in the existence of incentives and in the existence of opportunities increase, the assumed interrelationships become stronger, resulting in a higher risk of fraud than at lower levels and thus causing the shape of the graphs in Figure 6. Similar reasons can be attributed to the shape of the graphs in Figure 5.

Insert Figure 6 about here

RQ4: What is the impact on fraud risk of performing forensic audit procedures?

As noted previously, if fraud risk factors are identified, SAS No. 99 requires that the auditor decide on an appropriate response in regard to the audit program. With RQ4, we reexamine the scenario where the auditor finds no evidence suggesting that fraud may exist except for one or more factors related to a possible lack of management integrity. In response to this evidence, the auditor then performs forensic audit procedures specifically directed at the perceived exposure to fraud. As seen in Figure 7, when compared to Figure 3, fraud risk is reduced substantially for all levels of belief about management integrity.

Insert Figure 7 about here

IV. SUMMARY

In this paper we have developed a comprehensive analytical model for planning and evaluation of fraud risk [see Figure 1]. This model extends prior research (e.g., Srivastava and Shafer 1992, Dutta et al. 1998, and Turner et al. 2002) significantly by providing a closed-form calculation of fraud risk for both positive and negative evidence situations. Closed-form formulas that incorporate the fraud triangle factors (Wells 1997 and Montgomery et al. 2002), the impact of interactions on fraud risk due to their interrelationships and the impact of forensic procedures have also been derived in Appendices A and B. Figures 1 – 7 show the sensitivity of fraud risk to various combinations of these variables.

We have discussed several scenarios involving various combinations of audit evidence and have shown that fraud risk is likely to be underestimated if interrelationships among the fraud triangle factors are not considered (e.g., see Figure 3). As the strength of interrelationships increases, fraud risk increases exponentially. Thus, evaluations of categories of fraud risk factors should not be performed independent of one another. Otherwise, fraud risk will be underestimated leading to a potentially ineffective audit.

In the presence of fraud risk factors, belief in the effectiveness of mitigating factors must be at an extremely high level in order to reduce fraud risk to an acceptably low level (e.g., see Figure 2). Whether such high levels are feasible, even with strong internal controls, regulations, and laws, is an empirical question. These mitigating factors may reflect controls implemented internally by management or may arise externally due to the regulations such as the Sarbanes-Oxley Act of 2002 or due to other corporate governance regulations. However, most mitigating factors act only as deterrents to fraudulent financial reporting. As such, they do not provide strong evidence in support of the 'no fraud' assertion. Thus, under these assumptions, if fraud risk factors are present, the presence of mitigating factors alone may not be strong enough to reduce fraud risk to an acceptably low level and auditors need to perform appropriate forensic audit procedures to control fraud risk (e.g., see Figure 7).

While our formulation of the model incorporating the fraud triangle factors and the interrelationships is exact, we consider only separate and independent risk factors and mitigating factors for each factor in the fraud risk triangle. This assumption allows us to derive the fraud risk model in a closed form. For situations where one risk factor or mitigating factor pertains to two or more fraud risk categories, the evidential diagram becomes a network and a closed form of the model has not been derived. However, prior research shows that our model is a good, conservative approximation of the exact network model (Srivastava and Lu 2002).

There are many directions for future research in this area. For example, what analytical form would a full comprehensive audit risk model take if both fraud risk and risks due to unintentional errors are integrated? The models sketched here also need empirical research concerning the frequency and strength of fraud risk factors in practice (e.g., Mock and Turner 2001, 2003 and Graham and Bedard 2003) and on the effectiveness of both specific and general mitigating factors such as those included in the Sarbanes–Oxley Act of 2002. Also, to apply such models in practice, empirical evidence is needed on the strength of forensic procedures such as those suggested by SAS No. 99.

APPENDIX A

DERIVATION OF FRAUD RISK FORMULA WITH FRAUD TRIANGLE FACTORS

In this appendix we derive an analytical formula for fraud risk in terms of the risks that the following three fraud risk factors are present: existence of incentives, lack of management integrity, and existence of opportunities. In addition, we incorporate in the fraud risk model interrelationships between existence of incentives and lack of management integrity, and between lack of management integrity and existence of opportunities. The derivation process essentially involves combining the beliefs (m-values) obtained from various items of evidence pertaining to the three fraud components: incentives, integrity, and opportunities, as depicted in the evidential diagram in Figure 1. In other words, the process involves combining the level of assurance obtained from various items of evidence in favor of, against, or both for each of the components. We use the approach described by Srivastava (1995) to combine all the evidence.

There are three steps involved in the process. The first is to draw an evidential network for the problem by identifying the assertions and sub-assertions pertinent to the problem, identifying the relationships among these assertions and sub-assertions, and identifying items of evidence pertaining to these assertions and sub-assertions. The second step involves identifying the clusters of assertions (variables) over which we have belief functions either from the evidence or from the logical relationships and then creating a Markov tree of these clusters of variables. The third step involves propagating the belief functions (m-values) through the Markov tree to obtain the overall beliefs at the variable of interest. These steps are elaborated below:

Step 1: Draw the Evidential Diagram

Figure 1 represents an evidential diagram for determining if there is a risk of fraud based on the three fraud triangle components, 'No Incentives (NI)', 'Integrity (IT)', and 'No Opportunity (NO)'. Items of evidence pertaining to these components (variables) may be either risk factors or mitigating factors as described in SAS No. 99 (AICPA 2002) or as required by the Sarbanes-Oxley Act of 2002. The following steps can be used to draw an evidential diagram for the problem of interest (see Figure 1).

First, the main assertion, corresponding sub-assertions, and any sub-sub-assertions that may be needed must be identified (e.g., see Srivastava and Mock 1999-2000). These assertions are represented by the variable nodes shown as rounded boxes in the diagram. In the present case, the main assertion is 'No Fraud (NF)' as represented by the uppermost node in Figure 1.[8] Three sub-assertions representing the three fraud factors are: 'No Incentives (NI)', Integrity (IT)' and 'No Opportunities (NO)' and are shown in the lower nodes.

Next, these variable nodes (the main assertion and the sub-assertions) must be connected through logical relationships. In the present case, the three fraud triangle variable nodes, NI, IT,

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[8] We use upper case letters for the names of the variable nodes and lower case letters for their values. For example, 'NI' stands for 'No Incentive' where as 'ni' and '~ni', respectively, represent the values that the assertion is either true or false, i.e., 'ni' means there is no incentive and '~ni' means there is incentive (see Table 1 for details).

and NO, are connected through an 'OR' relationship to the node 'NF'.[9] This relationship implies that the management will commit fraud only when the three connected fraud factors exist: incentives to commit fraud, lack of management integrity, and opportunities are available to perpetrate fraud.

In the present case shown in Figure 1, we assume the variable 'No Incentive (NI)' is related to the variable 'Integrity (IT)' through a relationship r_1 that determines the degree of influence between incentives and the management integrity.[10] Note that the relationship r_1 is bidirectional in its influence. For example, in the case where there is no evidence pertaining to the variable 'Integrity', this relationship would influence the belief against management integrity if there is belief that management has incentives to commit fraud. Also, if there is evidence that there is no incentive to commit fraud, this relationship would influence the belief in favor of management integrity.

Conversely, if there is no evidence pertaining to the variable 'No Incentive', but there is belief that management lacks integrity, then the relationship would imply that management may

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[9] The belief function representation of the 'OR' relationship between 'NF' and the three fraud factor variables: NI, IT, and NO, is given by an m-value of 1 over all the possible values of the joint space (see Srivastava and Lu 2002 for such relationships):

m_{OR}({(nf,ni,it,no), (nf,ni,it,~no), (nf,ni,~it,no), (nf,~ni,it,no), (nf,ni,~it,~no), (nf,~ni,it,~no), (nf,~ni,~it,no), (~nf,~ni,~it,~no)}) = 1.

[10] The relationship r_1 is modeled in terms of the following m-values:

m_{R1}({(ni,it), (~ni,~it)}) = r_1, and m_{R1}({(ni,it), (ni,~it), (~ni,it), (~ni,~it)}) = 1- r_1.

create incentives. Furthermore, if there is evidence that management has strong integrity, this relationship would imply that management may take actions to negate incentives.

Similarly, we assume the variable ‘No Opportunity (NO)’ is related to the variable ‘Integrity (IT)’ through the relationship r_2. This relationship determines the degree of influence between the existence of opportunities and management integrity.[11] Again, this relationship is bidirectional in its influence and influences integrity if opportunities exist and influences the belief in opportunities if management lacks integrity.

Finally, the evidential diagram is completed by identifying various items of evidence pertaining to various assertions (variable nodes) and connecting them to the corresponding assertions. These items of evidence are represented as rectangular boxes. For example, as shown in Figure 1, evidence that there are ‘Mitigating Factors relevant to Incentives’ is represented by a rectangular box and is connected to node ‘IT’.

Step 2: Convert the Evidential Diagram into a Markov Tree

Srivastava (1995) and Shenoy (1991) define a Markov tree as a topological tree where nodes are subsets of variables. If a variable belongs to two distinct nodes, then every node lying on the path between these two nodes contains the variable. The properties of Markov trees and how to construct such trees have been studied and discussed by Kong (1986) and Mellouli (1987). Also, Markov trees are discussed in the computer science literature under the name "join trees" (e.g., see Maier 1983). Srivastava (1995) and Shenoy (1991) discuss in detail an algorithm

[11] The relationship r_2 is modeled in terms of the following m-values:

$$m_{R2}(\{(no,it), (\sim no,\sim it)\}) = r_2, \text{ and } m_{R2}(\{(no,it), (no,\sim it), (\sim no,it), (\sim no,\sim it)\}) = 1- r_2.$$

to construct a Markov tree from an evidential network. We will not discuss the algorithm here; rather we will use the approach to develop a Markov tree.

Insert Figure 8 about here

Figure 8 represents a Markov tree with eight nodes representing the evidential diagram given in Figure 1. As shown in Figure 8, four nodes (A, B, C, and H) have one variable, two nodes (D and E) have two variables, one node (F) has three variables, and one node (G) has four variables. Nodes D and E represent, respectively, the two relationships, r_1 and r_2. Node G represents an 'OR' relationship between 'NF' and the other three variables 'NI', 'IT' and 'NO'. As shown in Figure 1, there are items of evidence pertaining to the nodes, A, B, C. These items of evidence provide belief functions, i.e., m-values at the corresponding nodes. Nodes D and E, respectively, represent the relationships between 'NI' and 'IT', and between 'IT' and 'NO'. These relationships will define belief functions on nodes 'D' and 'E'.

Step 3: Propagation of Beliefs in a Markov Tree

The third step yields the overall belief at the main assertion, 'NF', after combining all the evidence. This is done by propagating m-values through the network (e.g., Srivastava 1995).[12]

[12] If m-values (i.e., the basic beliefs) are being propagated from a smaller node (having fewer variables) to a bigger node (having more variables) then one needs to vacuously extend the m-values on to the frame of the bigger node. For example, in propagating m-values from node A in Figure 8, which has one variable 'NI', a smaller node, with frame $\Theta_A = \{ni, \sim ni\}$, to a bigger node D with two variables, NI, and IT, and frame $\Theta_D = \{(ni,it), (ni,\sim it), (\sim ni,it), (\sim it, \sim ni)\}$, one has to vacuously extend the m-values defined on Θ_A to frame Θ_D. In other words, if

Column four of Table 3 shows the m-values propagated from node A to node D and from node C to node E.

Insert Table 3 about here

Next, as shown in Figure 8, m-values are propagated from nodes 'D', 'B' and 'E' to node F. Node F contains the following variables: NI, IT, and NO and its frame is:

$$\Theta_F = \{(ni,it,no), (ni,it,\sim no), (ni,\sim it,no), (ni,\sim it,\sim no), (\sim ni,it,no), (\sim ni,it,\sim no), (\sim ni,\sim it,no), (\sim ni,\sim it,\sim no)\}$$

The following m-values are the result of propagation from nodes B, D, and E to F:

<u>m-values from node B to node F</u>:

$$m_{B\to F}(\{(ni,it,no), (ni,it,\sim no), (\sim ni,it,no), (\sim ni,it,\sim no)\}) = m_{IT}^{+}$$

$$m_{B\to F}(\{(ni,\sim it,no), (ni,\sim it,\sim no), (\sim ni,\sim it,no), (\sim ni,\sim it,\sim no)\}) = m_{IT}^{-}$$

$$m_{B\to F}(\Theta_F) = m_{IT}^{\Theta}$$

<u>m-values from node D to node F</u>:

$$m_{D\to F}(\{(ni,it,no), (ni,it,\sim no)\}) = r_1\, m_{NI}^{+}$$

$$m_{D\to F}(\{(ni,it,no), (ni,it,\sim no), (ni,\sim it,no), (ni,\sim it,\sim no)\}) = (1 - r_1)\, m_{NI}^{+}$$

$$m_{D\to F}(\{(\sim ni,\sim it,no), (\sim ni,\sim it,\sim no)\}) = r_1\, m_{NI}^{-}$$

$$m_{D\to F}(\{(\sim ni,it,no), (\sim ni,it,\sim no), (\sim ni,\sim it,no), (\sim ni,\sim it,\sim no)\}) = (1 - r_1)\, m_{NI}^{-}$$

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one of the m-values, $m_A(ni) = m_{NI}^{+}$, when propagated to D, is vacuously extended to the bigger frame, the following m-value results: $m_{A\to D}(\{(ni, it), (ni, \sim it)\}) = m_{NI}^{+}$ (see Table 3 for details).

When m-values are propagated from a bigger node to a smaller node then the m-values are marginalized to the smaller node. The marginalization process in the belief-function framework is similar to the marginalization process in a probability framework.

$m_{D\rightarrow F}(\{(ni,it,no), (ni,it,\sim no), (\sim ni,\sim it,no), (\sim ni,\sim it,\sim no)\}) = r_1\, m^{\Theta}_{NI}$

$m_{D\rightarrow F}(\Theta_F) = (1 - r_1)\, m^{\Theta}_{NI}$

<u>m-values from node E to node F</u>:

$m_{E\rightarrow F}(\{(ni,it,no), (\sim ni,it,no)\}) = r_2\, m^{+}_{NO}$

$m_{E\rightarrow F}(\{(ni,it,no), (\sim ni,it,no), (ni,\sim it\ no), (\sim ni,\sim it\ no)\}) = (1 - r_2)\, m^{+}_{NO}$

$m_{E\rightarrow F}(\{(ni,\sim it,\sim no), (\sim ni,\sim it,\sim no)\}) = r_2\, m^{-}_{NO}$

$m_{E\rightarrow F}(\{(ni,it,\sim no), (\sim ni,it,\sim no), (ni,\sim it,\sim no), (\sim ni,\sim it,\sim no)\}) = (1 - r_2)\, m^{-}_{NO}$

$m_{E\rightarrow F}(\{(ni,it,no), (\sim ni,it,no), (ni,\sim it,\sim no), (\sim ni,\sim it,\sim no)\}) = r_2\, m^{\Theta}_{NO}$

$m_{E\rightarrow F}(\Theta_E) = (1 - r_2)\, m^{\Theta}_{NO}$

The m-values at node F is $m_F(\Theta_F) = 1$

We combine the above four m-values at node F by using Dempster's rule of combination (Shafer 1976). In this case, we must simplify a total of 108 (3 x 6 x 6) terms that result from multiplying the four m-values at node F with elements: 3, 6, 6, and 1. We simplify the process by combining two at a time. First, we combine $m_{D\rightarrow F}$ and $m_{E\rightarrow F}$ and then combine the resulting m-values with $m_{B\rightarrow F}$. The resulting m-values then are propagated to node G. Node G represents the logical relationship 'OR' between node 'NF' and the set of nodes: 'NI', 'IT', and 'NO'. There are two m-values at node G. One defines the 'OR' relationship and the other is propagated from node F, $m_{F\rightarrow G}$. The m-value defining the 'OR" relationship is given by:

$$m_G(\Theta_G) = m_G(\{(nf,ni,it,no), (nf,ni,it,\sim no), (nf,ni,\sim it,no), (nf,\sim ni,it,no), (nf,ni,\sim it,\sim no), (nf,\sim ni,it,\sim no), (nf,\sim ni,\sim it,no), (\sim nf,\sim ni,\sim it,\sim no)\}) = 1$$

The above two sets of m-values, $m_G(\Theta_G)$ and $m_{F\rightarrow G}$, are combined at G using Dempster's rule and then the resulting m-values are propagated to node 'NF'. Since node NF is a smaller node with frame Θ_{NF} = {nf, ~nf}, we need to marginalize the combined m-values at node G to the frame Θ_{NF} = {nf, ~nf} of node 'NF'. It is interesting to find that after simplifying 108 terms in the above process we obtain the following simple expressions for m-values at node 'NF':[13]

$$m_H(nf) = 1 - K^{-1}(m_{NI}^{\Theta} + m_{NI}^{-})(m_{IT}^{\Theta} + m_{IT}^{-})(m_{NO}^{\Theta} + m_{NO}^{-}) \tag{A1}$$

$$m_H(\sim nf) = K^{-1}[m_{IT}^{-}\{m_{NI}^{-} m_{NO}^{-} + r_1 m_{NI}^{\Theta} m_{NO}^{-} + r_2 m_{NI}^{-} m_{NO}^{\Theta} + r_1 r_2 m_{NI}^{\Theta} m_{NO}^{\Theta}\}$$
$$+ m_{IT}^{\Theta}\{(r_1 + r_2 - r_1 r_2) m_{NI}^{-} m_{NO}^{-} + r_1 r_2 (m_{NI}^{-} m_{NO}^{\Theta} + m_{NI}^{\Theta} m_{NO}^{-})\}] \tag{A2}$$

$$m_H(\Theta_H) = 1 - m_H(nf) - m_H(\sim nf) \tag{A3}$$

where K is given by:

$$K = 1 - m_{IT}^{+}(r_1 m_{NI}^{-} + r_2 m_{NO}^{-} - r_1 r_2 m_{NI}^{-} m_{NO}^{-}) - m_{IT}^{-}(r_1 m_{NI}^{+} + r_2 m_{NO}^{+} - r_1 r_2 m_{NI}^{+} m_{NO}^{+})$$
$$- r_1 r_2 m_{IT}^{\Theta}(m_{NI}^{+} m_{NO}^{-} + m_{NI}^{-} m_{NO}^{+}) \tag{A4}$$

The above m-values yield the following beliefs and plausibility for no fraud and fraud situations:

$$\text{Bel(no fraud)} = 1 - K^{-1}(m_{NI}^{\Theta} + m_{NI}^{-})(m_{IT}^{\Theta} + m_{IT}^{-})(m_{NO}^{\Theta} + m_{NO}^{-}) \tag{A5}$$

$$\text{Bel(fraud)} = K^{-1}\mathbf{[}m_{IT}^{-}\mathbf{\{}m_{NI}^{-} m_{NO}^{-} + r_1 m_{NI}^{\Theta} m_{NO}^{-} + r_2 m_{NI}^{-} m_{NO}^{\Theta} + r_1 r_2 m_{NI}^{\Theta} m_{NO}^{\Theta}\mathbf{\}}$$
$$+ m_{IT}^{\Theta}\mathbf{\{}(r_1 + r_2 - r_1 r_2) m_{NI}^{-} m_{NO}^{-} + r_1 r_2 (m_{NI}^{-} m_{NO}^{\Theta} + m_{NI}^{\Theta} m_{NO}^{-})\mathbf{\}]} \tag{A6}$$

and plausibility that there could be fraud is given by:

---

[13] We have checked our analytical expressions with various numerical input values of beliefs and the strength of relationships, r_1 and r_2, and compared the numerical output beliefs with the results obtained from a computer program, Auditor's Assistant, developed by Shafer et al. (1988) and find the two to be exactly the same.

$$Pl(fraud) = 1 - Bel(no\ fraud) = K^{-1}(m_{NI}^{\Theta} + m_{NI}^{-})(m_{IT}^{\Theta} + m_{IT}^{-})(m_{NO}^{\Theta} + m_{NO}^{-})$$

$$Pl(fraud) = Pl(\sim ni)Pl(\sim it)Pl(\sim no)/K \qquad (A7)$$

The above relationship can be written as:

$$Pl(fraud) = [Pl(Incentives)\ x\ Pl(Lack\ of\ Integrity)\ x\ Pl(Opportunities)]/K \qquad (A8)$$

APPENDIX B

EXTENSIONS OF FRAUD RISK FORMULAS IN TERMS OF RISKS DUE TO RISK FACTORS, MITIGATING FACTORS, AND FORENSIC PROCEDURES

There are two main objectives of this appendix. The first objective is to extend the general fraud risk formula in (A8) to include in specific terms the risks pertaining to the three fraud factors, presence of incentives, lack of integrity, and presence of opportunities, arising from the relevant internal and external risk factors and mitigating factors as discussed in the Introduction and Section III. The second objective is to further extend the fraud risk formula to incorporate the risk associated with forensic procedures.

Extension of Fraud Risk Formula to Incorporate Individual Risks

Here, we demonstrate that one can express plausibilities or risks in (A8) associated with the presence of incentives (~ni), lack of integrity (~it), and presence of opportunities (~no), in terms of specific risks as:

$$\text{Pl(Incentives)} = \frac{(R_{RF}^{NI})(R_{MF}^{NI})}{K_{NI}} \tag{B1}$$

$$\text{Pl(Lack of Integrity)} = \frac{(R_{RF}^{IT})(R_{MF}^{IT})}{K_{IT}} \tag{B2}$$

$$\text{Pl(Opportunities)} = \frac{(R_{RF}^{NO})(R_{MF}^{NO})}{K_{NO}} \tag{B3}$$

where R_{RF}^{NI}, R_{MF}^{NI}, R_{RF}^{IT}, R_{MF}^{IT}, R_{RF}^{NO} and R_{MF}^{NO} represent various plausibilities or risks associated with the presence of incentives, lack of integrity, and presence of opportunities, as defined in Table 1, arising from the corresponding risk factors (RF) and mitigating factors (MF). The symbols K_{NI}, K_{IT}, and K_{NO} are renormalization constants as defined below.

The proof of the relationship (B1) is detailed in the following discussion. Although not presented, this proof can be extended to the other two relationships.

Let us assume that for the variable NI, we have two items of evidence—one from the Risk Factors (RF) and the other from the Mitigating Factors (MF) (see Figure 1). Also, assume that the following basic belief masses (m-values) are obtained from the two items of evidence pertaining to 'NI':

m-values from RF: $m_{RF}(ni)$, $m_{RF}(\sim ni)$, and $m_{RF}(\{ni,\sim ni\})$.

m-values from MF: $m_{MF}(ni)$, $m_{MF}(\sim ni)$, and $m_{MF}(\{ni,\sim ni\})$.

We obtain the following m-values at the variable 'NI' after combining the above two m-values using Dempster's rule. In fact, these are the m-values assumed to be defined at NI in Figure 8 and used in A1-A8 in Appendix A.

$$m_{NI}(ni) = m^{+}_{NI} = [m_{RF}(ni)m_{MF}(ni) + m_{RF}(ni)m_{MF}(\{ni,\sim ni\}) + m_{RF}(\{ni,\sim ni\})m_{MF}(ni)]/K_{NI}$$

$$m_{NI}(\sim ni) = m^{-}_{NI} = [m_{RF}(\sim ni)m_{MF}(\sim ni) + m_{RF}(\sim ni)m_{MF}(\{ni,\sim ni\}) + m_{RF}(\{ni,\sim ni\})m_{MF}(\sim ni)]/K_{NI}$$

$$m_{NI}(\{ni,\sim ni\}) = m^{\Theta}_{NI} = [m_{RF}(\{ni,\sim ni\})m_{MF}(\{ni,\sim ni\})]/K_{NI}$$

where

$$K_{NI} = 1 - [m_{RF}(ni)m_{MF}(\sim ni) + m_{RF}(\sim ni)m_{MF}(ni)].$$

Similar expressions for K_{IT} and K_{NO} are defined in terms of the corresponding m-values.

The above m-values yield the following result for the Plausibility of '~ni':

$$\begin{aligned} Pl(Incentives) = Pl(\sim ni) &= [m_{NI}(\sim ni) + m_{NI}(\{ni,\sim ni\})] \\ &= [m_{RF}(\sim ni) + m_{RF}(\{ni,\sim ni\})][m_{MF}(\sim ni) + m_{MF}(\{ni,\sim ni\})]/ K_{NI} \\ &= Pl_{RF}(\sim ni)Pl_{MF}(\sim ni)/K_{NI} = \frac{(R^{NI}_{RF})(R^{NI}_{MF})}{K_{NI}} \qquad \text{(QED)} \end{aligned}$$

Now using the relationships in (B1-B3) we can rewrite the general fraud risk formula in (A8) as:

$$\text{Fraud Risk} = \text{Pl(Fraud)} = \frac{\left[\frac{(R_{RF}^{NI})(R_{MF}^{NI})}{K_{NI}} \text{ x } \frac{(R_{RF}^{IT})(R_{MF}^{IT})}{K_{IT}} \text{ x } \frac{(R_{RF}^{NO})(R_{MF}^{NO})}{K_{NO}}\right]}{K} \quad \text{(B4)}$$

Extension of Fraud Risk Formula to Incorporate Individual Risks and Risk due to Forensic Procedures

The fraud risk formula derived in (A8) or in (B4) was based on the assumption that we do not have any evidence bearing directly on the variable ‘NF’ or ‘H’ (see Figure 8). Thus, the overall m-values represented by m_H in (A1-A3) for this variable are due to the fraud triangle factors. For the current situation, we assume that we have performed forensic procedures on the relevant accounts and have obtained the following basic beliefs, i.e., m-values: $m_{FP}(nf)$, $m_{FP}(\sim nf)$, $m_{FP}(\{nf, \sim nf\})$.

In order to obtain the total belief or plausibility that there is no fraud or there is fraud, we use Dempster’s rule to combine the m-values in (A1-A3) with the m-values from the forensic procedures. This combination yields the following set of total m-values, m_T, at ‘No Fraud’ with frame $\Theta_{NF} = \{nf, \sim nf\}$:

$$m_T(nf) = [m_H(nf)m_{FP}(nf) + m_H(nf)m_{FP}(\Theta_{NF}) + m_H(\Theta_{NF})m_{FP}(nf)]/K_0, \quad \text{(B5)}$$

$$m_T(\sim nf) = [m_H(\sim nf)m_{FP}(\sim nf) + m_H(\sim nf)m_{FP}(\Theta_{NF}) + m_H(\Theta_{NF})m_{FP}(\sim nf)]/K_0, \quad \text{(B6)}$$

$$m_T(\{nf, \sim nf\}) = m_H(\Theta_{NF})m_{FP}(\Theta_{NF})/K_0, \quad \text{(B7)}$$

where

$$K_0 = 1 - [m_H(\sim nf)m_{FP}(nf) + m_H(nf)m_{FP}(\sim nf))]. \quad \text{(B8)}$$

The overall plausibility in fraud, Pl(~nf), is given by:

$$Pl_T(\sim nf) = \text{Fraud Risk} = [m_T(\sim nf) + m_T(\{nf, \sim nf\})]$$

$$= [m_H(\sim nf) + m_H(\Theta_{NF})][m_{FP}(\sim nf)\}+m_{FP}(\Theta_{NF})]/K_0 = Pl_H(\sim nf)Pl_{FP}(\sim nf)/K_0.$$

Using the value of $Pl_H(\sim nf)$ from (A7) one obtains the following expression for the overall plausibility of fraud:

$$Pl_T(\sim nf) = \text{Fraud Risk} = Pl(\sim ni)Pl(\sim it)Pl(\sim no)Pl_{FP}(\sim nf)/KK_0. \tag{B9}$$

However, the product of three plausibilities, Pl(~ni)Pl(~it)Pl(~no), can be expressed in terms of individual risks arising from various risk factors (RF) and mitigating factors (MF) as shown in (B1-B3). Thus, combining (B1-B3) with (B9), one obtains the following formula for the total fraud risk that includes all the fraud risk factors, their interrelationships, individual risk and mitigating factors relevant to the three fraud triangle factors, and the forensic procedures:

$$\text{Fraud Risk} = \text{Pl(Fraud)} = \frac{\left[\frac{(R_{RF}^{NI})(R_{MF}^{NI})}{K_{NI}} \text{x} \frac{(R_{RF}^{IT})(R_{MF}^{IT})}{K_{IT}} \text{x} \frac{(R_{RF}^{NO})(R_{MF}^{NO})}{K_{NO}}\right] \text{x} R_{FP}}{KK_0} \tag{B10}$$

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Figure 1: Evidential Diagram for Fraud Risk with Fraud Triangle Factors and Forensic Procedures

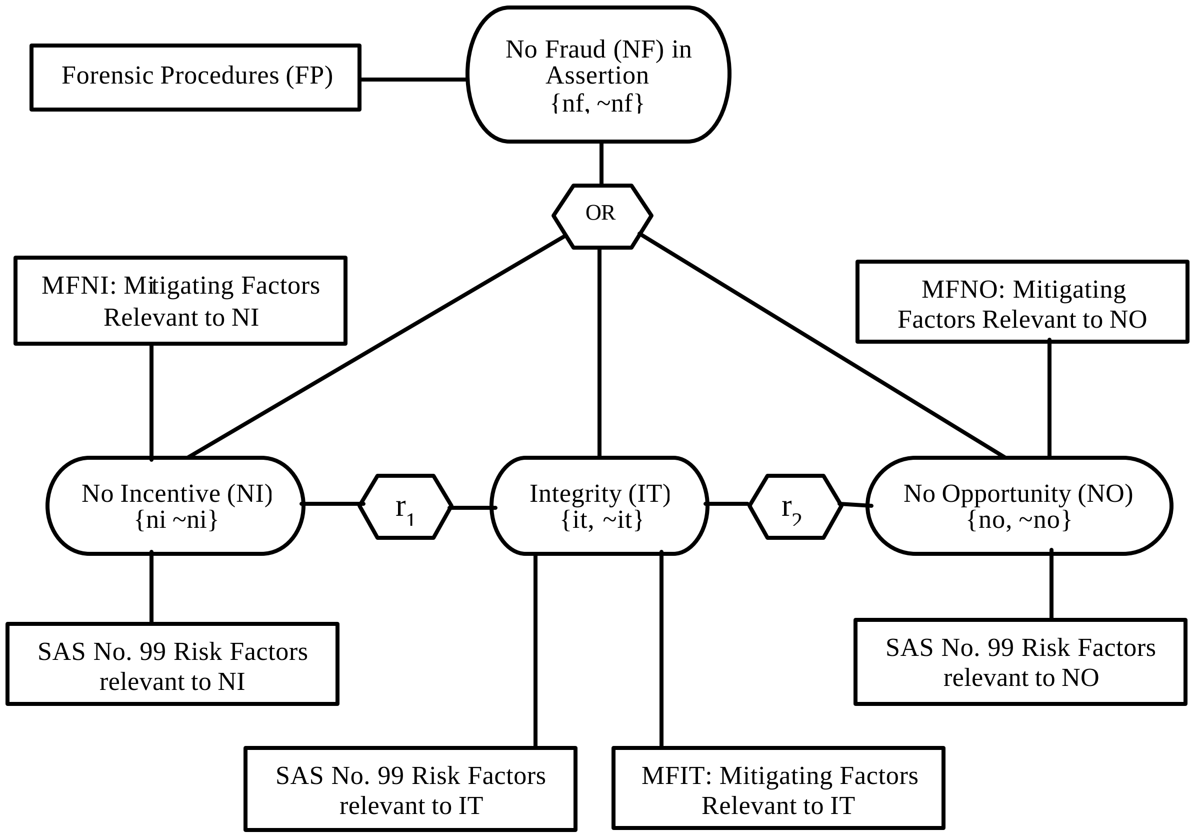
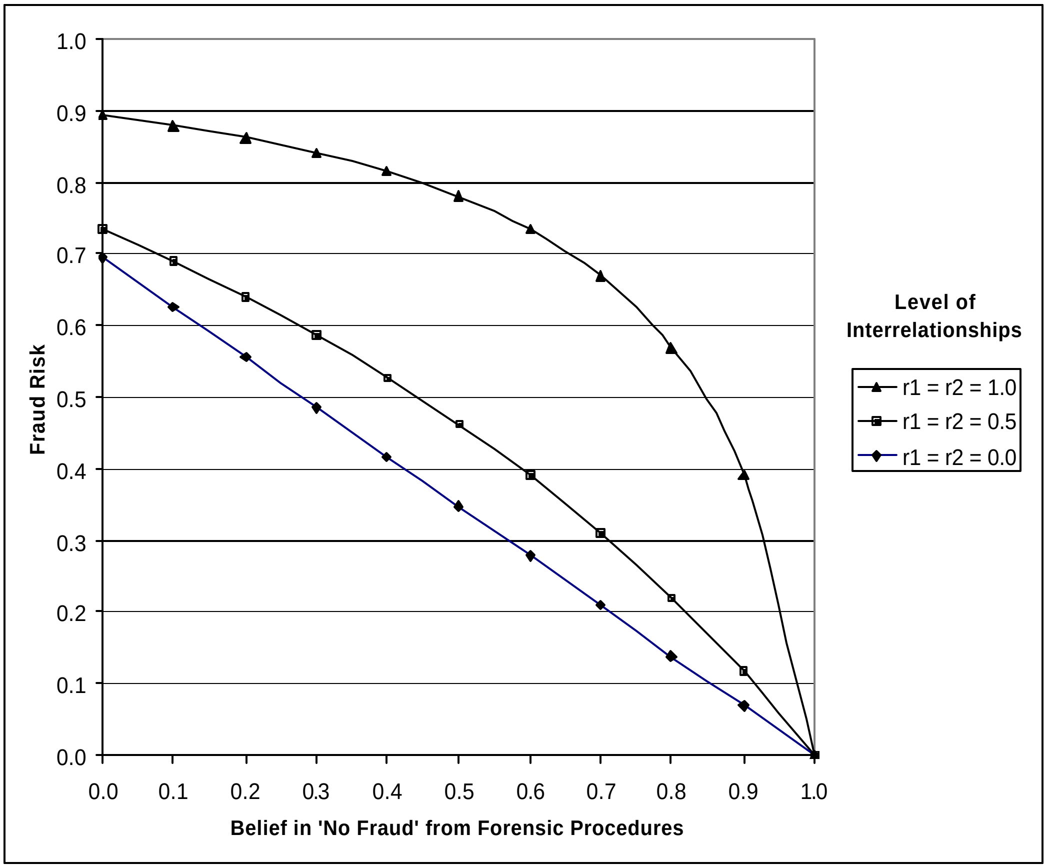
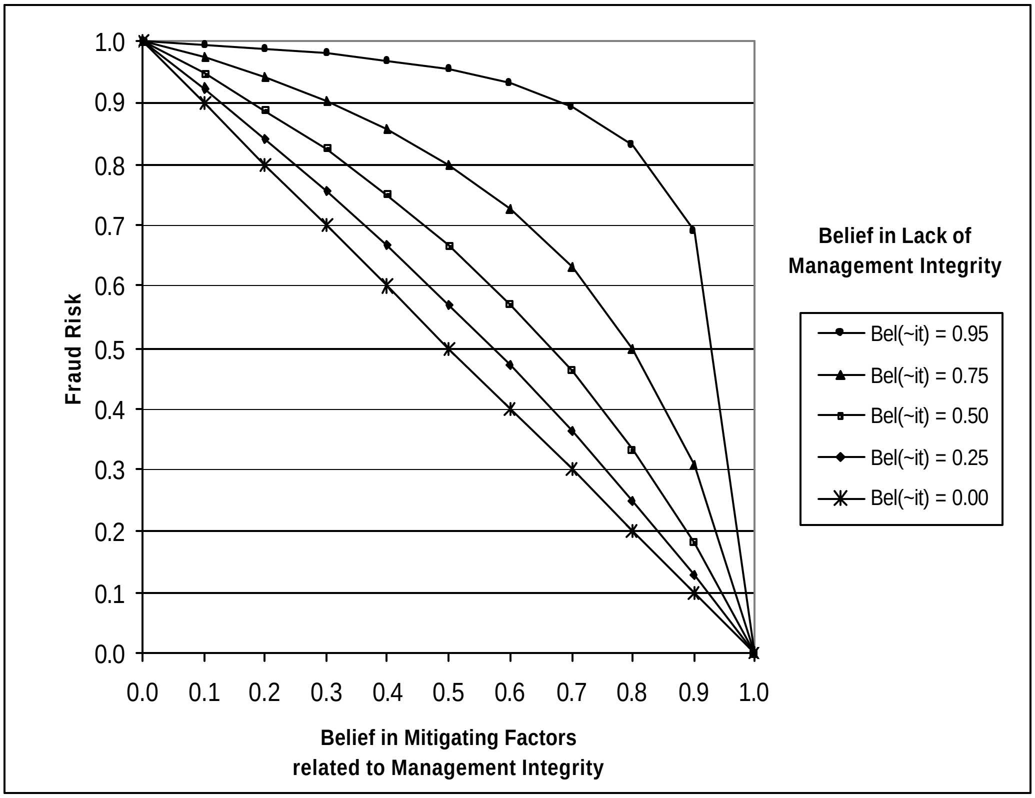


Figure 2: Fraud Risk as a Function of Belief in 'No Fraud' from Forensic Procedures for Various Strengths of Interrelationships, r_1 and r_2, among the Fraud Triangle Factors.*



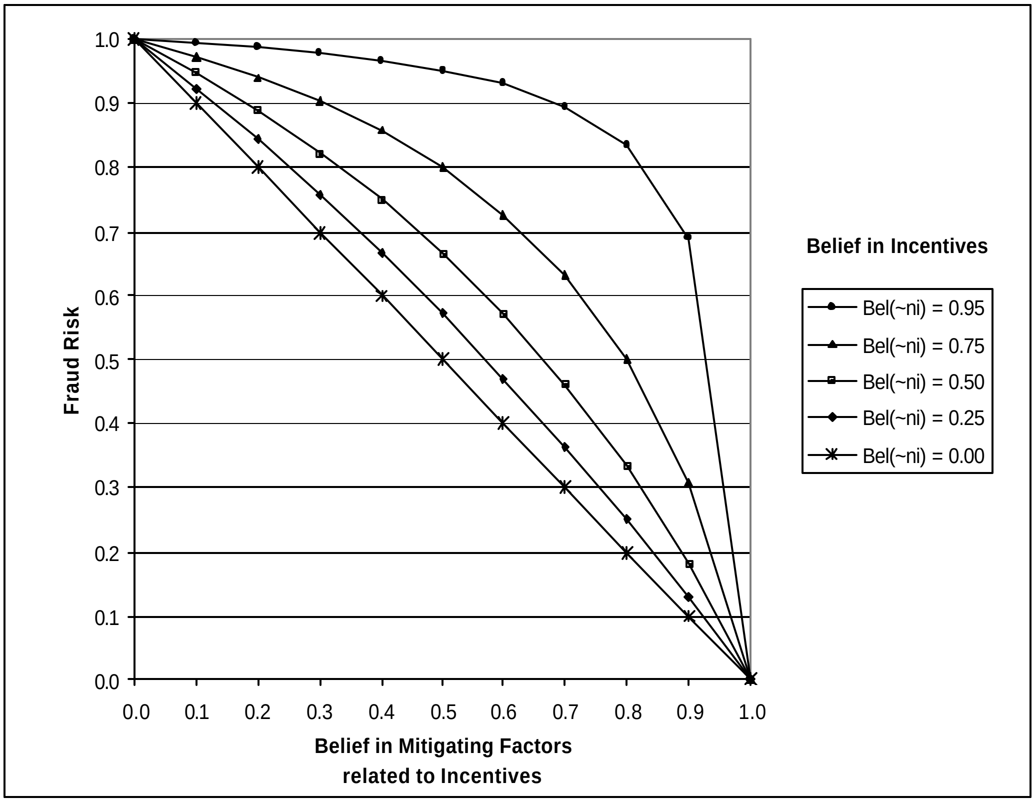
* See Table 2 for m-values used in this figure.

Figure 3: Fraud Risk as a Function of Belief in the Effectiveness of Mitigating Factors Associated with Management Integrity for Various Levels of Belief in a Lack of Management Integrity*



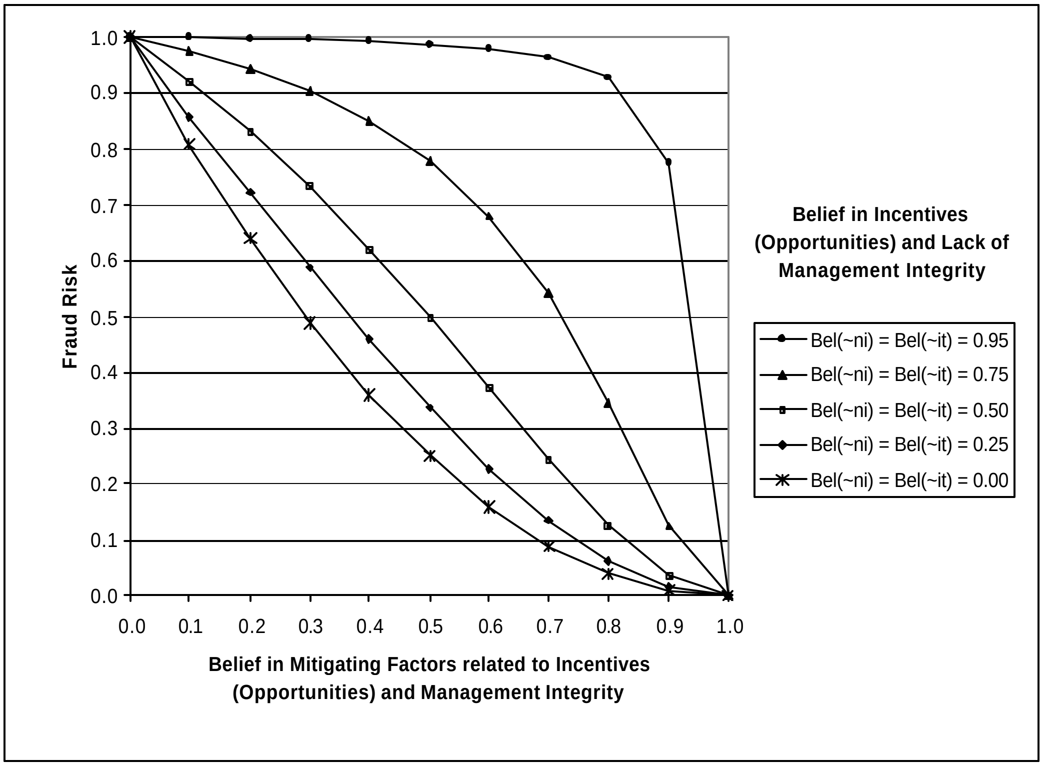
* See Table 2 for m-values used in this figure.

Figure 4: Fraud Risk as a Function of Belief in the Effectiveness of Mitigating Factors Associated with Incentives for Various Levels of Belief in the Existence of Incentives*



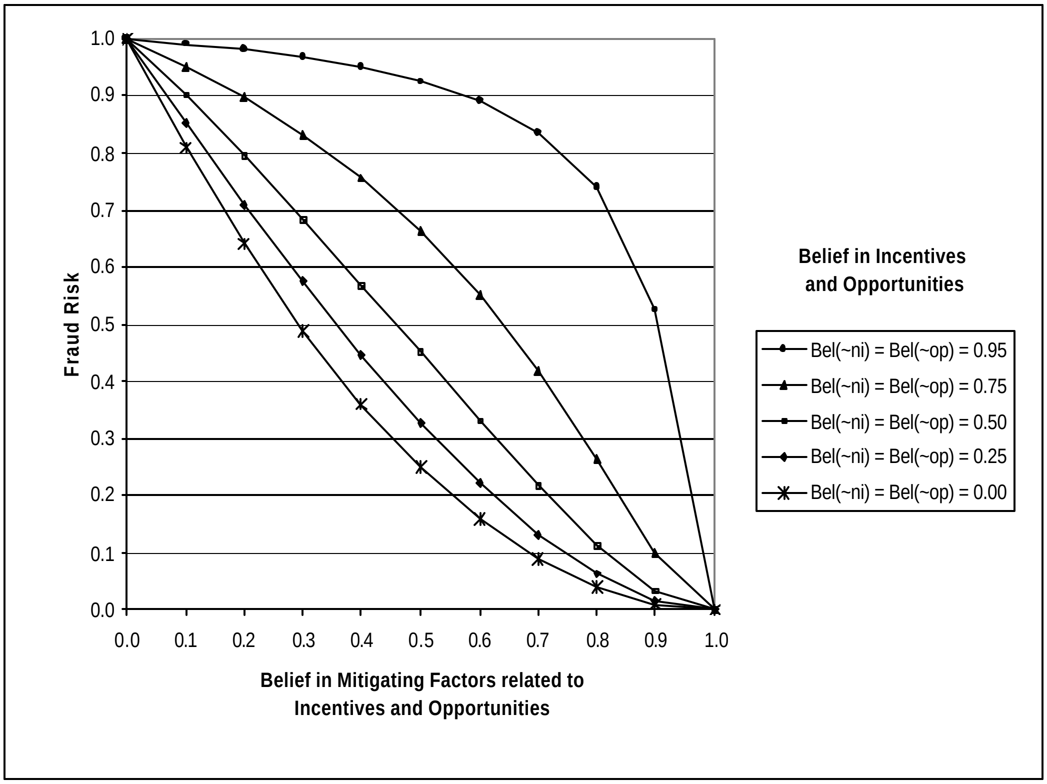
*See Table 2 for m-values used in this figure.

Figure 5: Fraud Risk as a Function of Belief in the Effectiveness of Mitigating Factors Associated with Incentives (Opportunities) and with Management Integrity for Various Levels of Belief in the Existence of Incentives (Opportunities) and Lack of Management Integrity*



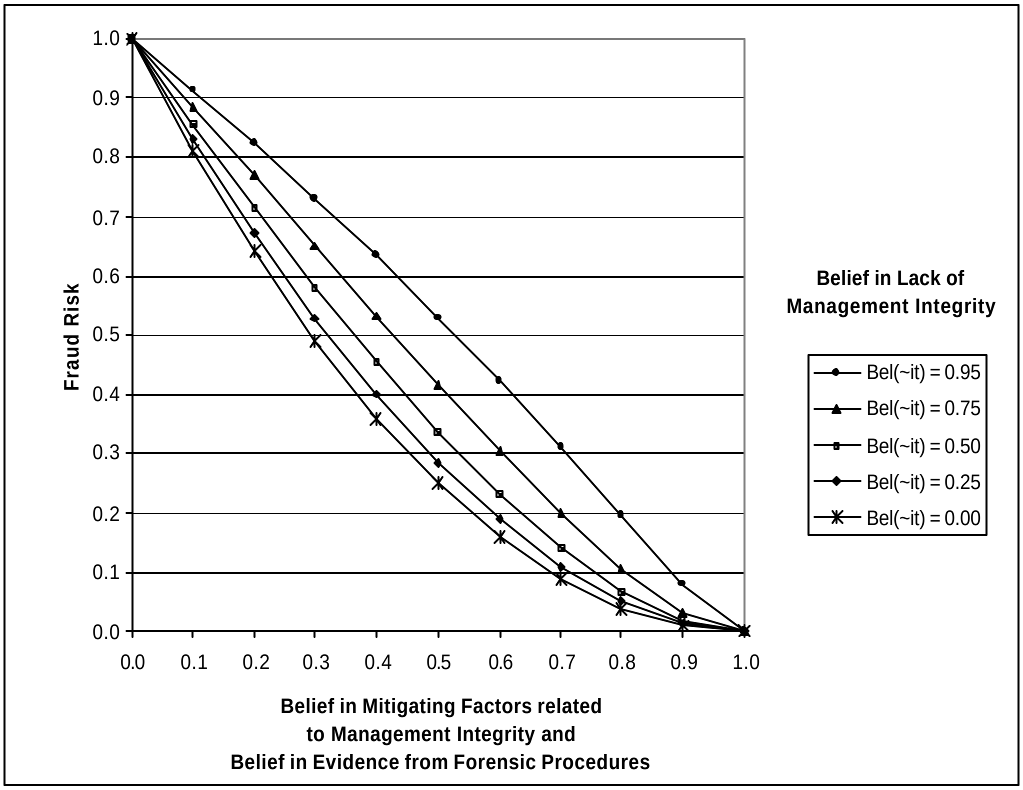
*See Table 2 for m-values used in this figure.

Figure 6: Fraud Risk as a Function of Belief in the Effectiveness of Mitigating Factors Associated with Incentives and with Opportunities for Various Levels of Belief in the Existence of Incentives and of Opportunities*



* See Table 2 for m-values used in this figure.

Figure 7: Fraud Risk as a Function of Belief in the Effectiveness of Mitigating Factors Associated with Management Integrity and with Evidence from Forensic Audit Procedures for Various Levels of Belief in the Lack of Management Integrity*



* See Table 2 for m-values used in this figure.

Figure 8: Markov Tree of the Evidential Diagram in Figure 1

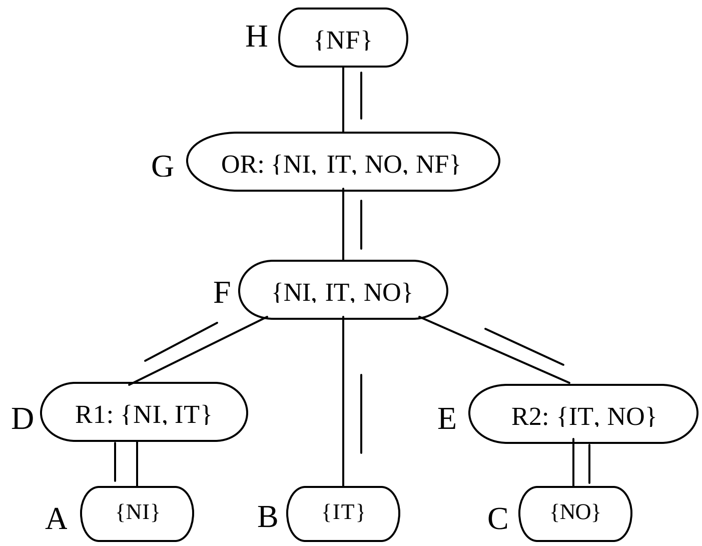


Table 1

List of Symbols and Their Descriptions

Symbol	Description
FP	Forensic procedures performed
IT	Variable representing 'Management Integrity'
MF	Mitigating Factors
NF	Variable representing 'No Fraud'
NI	Variable representing 'No Incentives'
NO	Variable representing 'No Opportunities'
RF	Risk Factors
it	Value of variable IT that it is true. i.e., 'it' represents that management has integrity.
~it	Value of variable IT that it is not true. i.e, '~it' represents that management lacks integrity.
nf	Value of NF that it is true. i.e, 'nf' represents that there is no fraud.
~nf	Value of NF that it is not true. i.e, '~nf' represents that there is fraud.
ni	Value of NI that it is true. i.e, 'ni' represents that there are no incentives.
~ni	Value of NI that it is not true. i.e, '~ni' represents that there are incentives.
no	Value of NO that it is true. i.e, 'no' represents that there are no opportunities.
~no	Value of NO that it is not true. i.e, '~no' represents that there are opportunities.
$m_{..}(..)$	The basic belief mass (m-value) for the value of the variable in the parenthesis from the evidence represented by the subscript.
$\Theta_{..}$	This symbol represents the frame of a variable denoted by the subscript. For example, the frame of variable 'NO' is represented as $\Theta_{NO} = \{no, \sim no\}$.
R_{RF}^{IT}	This symbol represents plausibility, $Pl_{RF}(\sim it)$, or risk based on risk factors (RF) that the management lacks integrity.
R_{MF}^{IT}	This symbol represents plausibility, $Pl_{MF}(\sim it)$, or risk based on mitigating factors (MF) that the management lacks integrity.
R_{RF}^{NI}	This symbol represents plausibility, $Pl_{RF}(\sim ni)$, or risk based on risk factors (RF) that there are incentives.
R_{MF}^{NI}	This symbol represents plausibility, $Pl_{MF}(\sim ni)$, or risk based on mitigating factors (MF) that there are incentives.
R_{RF}^{NO}	This symbol represents plausibility, $Pl_{RF}(\sim no)$, or risk based on risk factors (RF) that there are opportunities.
R_{MF}^{NO}	This symbol represents plausibility, $Pl_{MF}(\sim no)$, based on mitigating factors (MF) that there are opportunities.
R_{FP}	This symbol represents plausibility, $Pl_{FP}(\sim nf)$, or risk based on forensic procedures (FP) that fraud exists.

Table 2

Belief Values for Various Figures

Factors for which Evidence Gathered	Figure 2[1]	Figure 3[2]	Figure 4[2]	Figure 5[2]	Figure 6[2]	Figure 7[2]
Incentive/ Pressure Fraud Risk Factors $\{m_{RF}(ni), m_{RF}(\sim ni)\}$	{0, 0.8}	{0, 0}	{0,Varied*}	{0, Varied*}	{0, Varied**}	{0, 0}
Mitigating Factors related to Incentive/Pressure $\{m_{MF}(ni), m_{MF}(\sim ni)\}$	{0.5, 0}	{0, 0}	{Varied**, 0}	{Varied**, 0}	{0,Varied*}	{0, 0}
Management Integrity Fraud Risk Factors $\{m_{RF}(it), m_{RF}(\sim it)\}$	{0, 0}	{0,Varied*}	{0, 0}	{0, Varied*}	{0, 0}	{0,Varied**}
Mitigating Factors related to Integrity $\{m_{MF}(it), m_{MF}(\sim it)\}$	{0, 0}	{Varied**, 0}	{0, 0}	{Varied**, 0}	{0, 0}	{0, Varied*}
Opportunity Fraud Risk Factors $\{m_{RF}(no), m_{RF}(\sim no)\}$	{0, 0.8}	{0, 0}	{0, 0}	{0, 0}	{0, Varied**}	{0, 0}
Mitigating Factors related to Opportunity $\{m_{MF}(no), m_{MF}(\sim no)\}$	{0.5, 0}	{0, 0}	{0, 0}	{0, 0}	{0,Varied*}	{0, 0}
Forensic Procedures to Detect Financial Statement Fraud $\{m_{FP}(nf), m_{FP}(\sim nf)\}$	{Varied**, 0}	{0, 0}	{0, 0}	{0, 0}	{0, 0}	{0, Varied*}

[1] $r_1 = r_2 = 0.0, 0.5, \text{and } 1.0$

[2] $r_1 = \frac{m_{RF}(\sim ni) + m_{RF}(\sim it)}{2}$; $r_2 = \frac{m_{RF}(\sim it) + m_{RF}(\sim no)}{2}$, where $m_{RF}(\sim ni)$, $m_{RF}(\sim it)$, and $m_{RF}(\sim no)$, represent the m-values from risk factors for the corresponding values in the argument..

* Varied at values of 0.00, 0.25, 0.50, 0.75 and 0.95

** Varied from 0.0 to 1.0 in increments of 0.1

Table 3

Propagation of m-values from Node A to Node D and from Node C to Node E in the Markov tree in Figure 8
Only non-zero m-values are given in this table.

Node and variables	Frame of the node	m-values defined at the node	m-values at the node from the neighboring node(s)	Total m-values at the node (Obtained from combining m-values in columns 3 and 4 using Dempster's rule)
A = {NI}	Θ_A = {ni, ~ni}	$m_A(ni) = m_{NI}^{+}$ $m_A(\sim ni) = m_{NI}^{-}$ $m_A(\Theta_A) = m_{NI}^{\Theta}$	none	$m^t_A(ni) = m_{NI}^{+}$ $m^t_A(\sim ni) = m_{NI}^{-}$ $m^t_A(\Theta_A) = m_{NI}^{\Theta}$
D = {NI, IT} Relation: R1	Θ_D = {(ni, it), (ni, ~it), (~ni, it), (~ni, ~it)}	$m_D(\{(ni, it), (\sim ni, \sim it)\}) = r_1$ $m_D(\Theta_D) = 1 - r_1$	$m_{A\rightarrow D}(\{(ni, it), (ni, \sim it)\}) = m_{NI}^{+}$ $m_{A\rightarrow D}(\{(\sim ni, it), (\sim ni, \sim it)\}) = m_{NI}^{-}$ $m_{A\rightarrow D}(\Theta_D) = m_{NI}^{\Theta}$	$m^t_D(\{(ni, it)\}) = r_1\, m_{NI}^{+}$ $m^t_D(\{(ni, it), (ni, \sim it)\}) = (1 - r_1)\, m_{NI}^{+}$ $m^t_D(\{(\sim ni, \sim it)\}) = r_1\, m_{NI}^{-}$ $m^t_D(\{(\sim ni, it), (\sim ni, \sim it)\}) = (1 - r_1)\, m_{NI}^{-}$ $m^t_D(\{(ni, it), (\sim ni, \sim it)\}) = r_1\, m_{NI}^{\Theta}$ $m^t_D(\Theta_D) = (1 - r_1)\, m_{NI}^{\Theta}$
C = {NO}	Θ_C = {no, ~no}	$m_C(no) = m_{NO}^{+}$ $m_C(\sim no) = m_{NO}^{-}$ $m_C(\Theta_C) = m_{NO}^{\Theta}$	none	$m^t_C(no) = m_{NO}^{+}$ $m^t_C(\sim no) = m_{NO}^{-}$ $m^t_C(\Theta_C) = m_{NO}^{\Theta}$
E = {IT, NO} Relation: R2	Θ_E = {(it, no), (it, ~no), (~it, no), (~it, ~no)}	$m_E(\{(it, no), (\sim it, \sim no)\}) = r_2$ $m_E(\Theta_E) = 1 - r_2$	$m_{C\rightarrow E}(\{(it, no), (\sim it, no)\}) = m_{NO}^{+}$ $m_{C\rightarrow E}(\{(it, \sim no), (\sim it, \sim no)\}) = m_{NO}^{-}$ $m_{C\rightarrow E}(\Theta_E) = m_{NO}^{\Theta}$	$m^t_E(\{(it, no)\}) = r_2\, m_{NO}^{+}$ $m^t_E(\{(it, no), (\sim it, no)\}) = (1 - r_2)\, m_{NO}^{+}$ $m^t_E(\{\sim it, \sim no)\}) = r_2\, m_{NO}^{-}$ $m^t_E(\{(it, \sim no), (\sim it, \sim no)\}) = (1 - r_2)\, m_{NO}^{-}$ $m^t_E(\{(it, no), (\sim it, \sim no)\}) = r_2\, m_{NO}^{\Theta}$ $m^t_E(\Theta_E) = (1 - r_2)\, m_{NO}^{\Theta}$