

**The Impact of Judgment Enhancement Strategies on
Audit Quality and Firm Risk When Clients Have Correlated Business Risks**

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September 10, 2006

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We wish to thank the workshop participants at the University of Illinois, Michael Gibbins, Jonathan Grenier (research assistant), Mark Peecher, Bill Scott, Dan Simunic, and Ira Solomon, for their useful comments. Financial support from KPMG LLP and Foundation is gratefully acknowledged.

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Abstract

Public accounting firms face many challenges beyond control of audit risk when performing audits for multiple clients. These challenges stem from clients' correlated business risks and auditors' correlated judgment errors. An important strategy for acquiring knowledge and disseminating/leveraging expertise across audit engagements as well as dealing with clients' correlated business risks, while managing the firm's overall risk as a whole, is to develop "specialists" who can work on multiple engagements. Such auditor specialization potentially helps to reduce audit judgment errors and to induce greater uniformity in auditing practice across multiple engagements. However, even specialists may make judgment errors. When they engage multiple clients that have correlated business risks, the correlation among their judgment errors may outweigh their judgment precision improvement, resulting in higher firm risk. Linking the covariance of auditors' judgment errors explicitly with the covariance of clients' business risks provides new insight into auditor specialization and audit staffing policies.

Key Words: auditor specialization, audit quality and risk, correlated client-business risks, correlated audit-judgment errors

JEL Descriptors: M4, L15, G19

1. Introduction

Public accounting firms face a variety of challenges as well as opportunities, when they perform audits for multiple clients. Perhaps the most important challenges are audit quality control and firm risk management. While extant professional standards provide detailed guidance in dealing with quality/risk issues at the *engagement* level, accounting firms have great self-responsibility for developing their own strategies to address these issues systematically at the *firm* level (Bell *et al.* 1997, 2005).¹ Maintaining audit quality and managing risk at the firm level, rather than at the engagement level, encounter two complications that arise concurrently and interdependently. One of these complications is that clients are unlikely to have *independent* business or entity risks, particularly when they have economic/business relationships.² Thus, when assessing its clients' business risks, an accounting firm must consider its clients as a portfolio due to its ability, or lack of such ability, to diversify its clients' business risks (Simunic and Stein 1990).

The other complication is that any audit judgment errors are unlikely to be *independent*. Auditors within a firm often employ similar audit approaches and technologies, as well as consult with the same specialists. Thus, they acquire knowledge and disseminate/leverage the

¹ The Audit Standards Committee has issued several "Statements on Quality Control (QC) Standards" to deal with quality/risk control at the firm level. However, the standards are much less detailed than Statements on Auditing Standards or the PCAOB's recent Audit Standards and, thus, give firms considerably more discretion than at the engagement level. For example, QC Section 20.03 states that: "A firm has a responsibility to ensure that its personnel comply with the professional standards applicable to its accounting and auditing practice. A system of quality control is broadly defined as a process to provide the firm with reasonable assurance that its personnel comply with applicable professional standards and the firm's (own) standards of quality." The same level of generality is evident in QC Section 20.04 which states that, "The nature, extent, and formality of a firm's quality control policies and procedures should be appropriately comprehensive and suitably designed in relation to the firm's size, the number of its offices, the degree of authority allowed its personnel and its offices, the knowledge and experience of its personnel, the nature and complexity of the firm's practice, and appropriate cost-benefit considerations."

² While the authoritative professional auditing literature defines inherent risk, control risk, and detection risk rather precisely (see AU Section 312), client business risk has not been defined with a comparable level of specificity. Consistent with Simunic and Stein (1990), Bell *et al.* (1997), and Beck and Wu (2006), we use the terms of client business risk and client entity risk interchangeably (see the model section for details).

expertise from the specialists to the many audit teams within the firm across multiple audit engagements. To the extent that these judgment-enhancement strategies lead to expertise, auditors can improve the accuracy of their audit judgments and induce greater uniformity in their judgments across audit engagements (Libby and Luft 1993; Solomon, Shields and Whittington 1999). Nonetheless, such strategies could potentially induce a positive, high correlation among audit judgment errors (Gibbins 1982, 2000), which in turn may offset the accuracy improvement in judgments, resulting in lower audit quality and higher overall risk of the firm as a whole. Consequently, maintaining audit quality and managing firm risk at the firm level with multiple audit clients, an accounting firm must systematically consider the covariance among its clients' business risks, the covariance among its auditors' judgment errors, and their interaction, in addition to those at the engagement level.

Quality control and risk management strategies used to enhance judgments are many.³ However, an important strategy for acquiring, disseminating, and leveraging expertise across audit engagements as well as dealing with clients' correlated business risks, while maintaining audit quality and managing risk of the firm as a whole, is to develop "specialists" who can work

³ In accounting literature, many studies examine the flow of knowledge and information between audit engagements through industry and technical experts (Solomon, Shields and Whittington 1999; Taylor 2000; Owsho, Messier and Lynch 2002; Low 2004; and Hammersley 2005). Further, the important role of auditor specialization is underscored by the auditing professional standards requiring specialists to be considered as part of the audit planning process. The reliance on specialists has also raised other important issues such as audit staffing (Messmer 1992; O'Keefe, Simunic and Stein 1994; Prawitt 1995; Hackenbrack and Knechel 1997; AOMAR Survey 2005), audit process improvements (Cushing *et al.* 1995), client acceptance decisions (Johnstone and Bedard 2003), and audit fees (Simunic 1980; Simunic and Stein 1987; Bell, Landsman and Shackelford 2001; Ferguson and Stokes 2002; Lyon and Maher 2005).

Another knowledge-sharing strategy for audit judgment enhancement is to encapsulate the knowledge of experts into audit manuals, training programs, and decision aids (Graham, Damens and Nan Ness 1991; Gillett 1993; Bell *et al.* 2002). Cushing and Loebbecke (1986) and Kinney (1986) characterize firms that rely heavily on the encapsulation strategy as representing a "structured" approach to auditing. It should be noted that, irrespective of whether accounting firms rely on specialists, these structural features of the audit environment will induce a positive level of correlation in audit judgment errors (despite the reduction in the variability in judgment errors).

Alternatively, knowledge and information also flows between audit and non-audit services at the engagement level, through cost savings (Simunic 1984; Antle and Demski 1991), oligopolistic market interactions (Wu 2006), and learning by doing over time (Beck and Wu 2006).

on multiple audit engagements. Such auditor specialization potentially helps to facilitate knowledge creation and, thereby, improves judgment precision both at the engagement level and at the firm level.⁴ Although assigning specialists to participate on multiple audit engagements potentially improves audit quality, it can also induce high correlations in audit judgment errors. Correlations among auditors' judgment errors can interact with the correlations among clients' business risks, resulting in unintended, potentially adverse consequences.

In special cases where clients' business risks are *negatively* correlated and a specialist's judgment errors are *independent*, specialization has the intuitive and desirable consequences of reducing the firm's overall risk and enhancing audit quality. In practice, audit judgment errors are likely to be *positively* correlated, however, particularly when the same professional makes the judgments for multiple audit engagements.⁵ Furthermore, many reasons exist for clients' business risks to be positively, rather than negatively, correlated (Shank and Murdock 1978). One reason is that accounting firms have been developing concentrations in client industry groups.⁶ Another reason is that audit clients may have economic and business relationships due to supply-chain partnerships, strategic alliances, or even belong to the same consolidated group.

When clients' business risks are positively correlated, auditor specialization can either enhance or diminish audit quality, depending on its effect on the covariance of judgment errors across these engagements. While specialization helps to reduce the variability in judgment errors

⁴ Davis and Solomon (1989) emphasize that developing expertise requires more than the mere acquisition of knowledge and experience. Expertise is outcome-based and requires attributes such as ability, motivation, and environment in addition to knowledge (Libby and Luft 1993). However, we use these terms interchangeably. Auditor specialization may be based technology, accounting, or business issues related to an industry.

⁵ It is noteworthy that the PCAOB's inspection process is predicated on the presence of judgment-error correlations. When PCAOB inspectors find evidence of deficiencies, they immediately inspect the audit working papers of other engagements supervised by the same partners and managers.

⁶ The literature on industry concentration is extensive (Eichenseher and Danos 1981; Danos and Eichenseher 1986; Ferguson, Francis and Stokes 2003; Deltas and Doogar 2002, 2004; Francis, Reichelt and Wang 2005; a review by Bedard, Mock and Wright 1999). Others also use the portfolio approach to consider the audit clientele effect (Choi, Doogar and Ganguly 2004; Johnstone and Bedard 2004).

at the engagement level, a specialist working on multiple engagements is also likely to propagate his or her judgment errors across multiple engagements at the firm level. Since the covariance is dependent on both the correlation and variance components, the ultimate effect on the firm's overall risk exposure depends on which component predominates. Further, the consequences of specialization depend also on the magnitude of the auditors' judgment-error correlation *relative to* that of the clients' business-risk correlation: Auditor specialization impacts audit quality in different ways, depending on how and the extent to which clients' business risks are correlated.

The multi-client model, when applied to affiliated companies, provides several intriguing implications for auditing practice. First, affiliated companies represent one extreme example that clients have correlated business risks due to the application of the "equity" accounting method for investments and the consolidation process. A second feature of audits of affiliated companies concerns the extreme diversity among accounting firms' staffing policies. At one Big 4 firm, for example, the same audit team audits all affiliated domestic companies whenever possible; so we would expect audit judgment errors to be maximally correlated. At another Big 4 firm, auditors of parent companies delegate the audit staffing of the subsidiaries to teams from the office in closest geographical proximity. Thus, we would expect audit judgment errors to be minimally correlated.⁷ Consequently, the tradeoff between judgment-precision enhancement and judgment-error-correlation inducement helps to reconcile these seemingly conflicting staffing policies for audits of affiliated companies.⁸

⁷ Audit structure (Cushing and Loebbecke 1986, Kinney 1986) could also be a contributing factor. The second Big 4 firm is comprised of two former Big-8 firms that were respectively classified as having "moderately" and "highly" structured audit approach, while the first Big 4 firm is comprised of two former Big-8 firms that were both classified as having "low" levels of audit structure. Since the combined firm has made significant changes in its audit approaches in recent years, the Cushing and Loebbecke (1986) classification would no longer be descriptive.

⁸ At the second Big 4 firm, partners and managers are highly restricted in terms of their ability to preference audit team members who are typically rotated across client industry groups. Hence, the ability of partners to reduce judgment errors through the staff-preferencing process would be dramatically constrained, thus, explaining why the firm's strategy of assigning affiliates to different audit teams might be effective at reducing the correlation problem.

The remaining sections of the paper are as follows. Section 2 describes the basic model of a public accounting firm. Section 3 introduces the surrogate for audit quality and, hence, the value of knowledge sharing between audit engagements. Section 4 presents comparative static analyses of audit quality with respect to both “correlation” and “precision” parameters of clients’ business risks as well as auditors’ judgment errors. In Section 5, we study audit staffing policies as a specific example. Since staffing policies are likely to affect both the precision and the correlation of audit judgments, their overall impact on audit quality is generally complex. Section 6 contains concluding remarks and implications. All proofs are in the appendix.

2. The model

We present a stylized model of an accounting firm that services two audit clients (as depicted in Figure 1).⁹ Each client has an earnings distribution — a summary statistic for the client’s industry, capital structure, business strategy, operations, and controls — that is denoted by \tilde{x}_i (for $i = 1, 2$) and assumed to be normally distributed with mean \bar{x}_i , variance $1/s_i$, and correlation $\gamma \equiv \text{corr}(\tilde{x}_1, \tilde{x}_2)$ (where “ \sim ” denotes random variables). The variance, $1/s_i$, is determined by both the industry and client-specific factors related to the complexity of the client’s operations and controls. Collectively, we refer to these factors as the client’s “earnings-generation process.”

The earnings variance, however, is not the only risk-driver in the two-client model. The clients’ earnings distributions may be correlated and, hence, co-vary with each other (Simunic and Stein 1990). Thus, another risk-driver in the model is the correlation between the clients’

⁹ The extension to an accounting firm with more than two audit clients is straightforward, and we leave the extension to multiple periods for future research (see Morgan and Stocken 1998; Beck and Wu 2006 for analysis of the auditor’s learning about his or her single client over time). Gibbins (1982) has considered the impact of inter-correlated data sources on auditor judgment. Such data sources can be thought of as the audit engagements in our model. Thus, the inter-correlation may arise from the clients’ business risks and/or the auditors’ judgment errors.

business risks, $\gamma \in [-1, +1]$ (see Shank and Murdock 1978). Its sign and magnitude depend on the economic relationships, if any, among the clients, and the underlying market structure. Due to economic and industry factors, we would expect the sign to be positive, and the magnitude to depend on client specifics. For example, if the clients are partners in a supply-chain relationship or are business alliances, then γ is likely to be positive and increasing in proportion to the importance of their business relationships. Perhaps the extreme example is when one client is an affiliate of another, so γ could be close to one. Alternatively, if clients compete in a stable, oligopolistic industry (wherein they can expand only by taking business away from competitors), then γ could be negative.¹⁰ Therefore, we can think of the absolute value $|\gamma|$ as a measure of the strength of the association between the clients' economic and/or business environments.

Each audit engagement requires numerous professional judgments, so that the audit-reporting decisions will be based on a coherent integration of these auditor judgments. In the model, we abstract from these details that are involved in the decision-making process of audit judgments (Cushing and Loebbecke 1986, Chapter 4; Kinney 1986; Morris and Nichols 1988). Specifically, we assume that the audit judgments are made about each client's earnings and that these judgments are imperfect (i.e., noisy). We represent the auditor's assessment of the client's earnings distribution by:

$$\tilde{y}_i = \tilde{x}_i + \tilde{\omega}_i, \quad (1)$$

where $\tilde{\omega}_i \sim N(0, 1/h_i)$ and $h_i > 0$ (for $i = 1, 2$).

¹⁰ One dramatic example is the negative correlation in earnings between K-Mart and Walmart during the past two decades. Regressing K-Mart's quarterly sales revenues (for the period 1988 to 2000) onto Walmart's quarterly sales yields highly significant, negative coefficient. This result is robust after controlling for various K-Mart store counts, macro-economic variables, as well as seasonality effects. Earnings regressions indicated a similar relationship — although the explanatory power is lower due to the volatile earnings of K-Mart over that time period.

The judgment-error term, $\tilde{\omega}_i$, in (1) represents the imperfection inherent in the auditor’s information about the client’s earnings distribution (see Libby and Luft 1993 for reasons that even an expert cannot extract all errors from the judgment process). *Ceteris paribus*, auditors having specialized knowledge should be able to assess “tighter” earnings distributions (larger h_i) than those assessed by auditors lacking such specialized knowledge.¹¹ In addition, without loss of generality, we assume that $\text{cov}(\tilde{x}_i, \tilde{\omega}_i) = 0$ and $\text{cov}(\tilde{x}_i, \tilde{\omega}_j) = 0$, for $i, j = 1, 2$; and $i \neq j$.¹²

We do admit the possibility that the judgment errors across the audit engagements are nontrivially correlated with each other: $\rho \equiv \text{corr}(\tilde{\omega}_1, \tilde{\omega}_2) \neq 0$. We would expect that $\rho > 0$, in general, due to the presence of auditing professional standards. The magnitude of ρ will increase with the degree of structure inherent in the firm’s audit methodology, and with the actual overlap of staff members who are involved in forming audit judgments. That is, ρ is a surrogate for the degree of “uniformity” inherent in audit judgment errors.¹³

Under the above assumptions, the covariance between the two audit engagements will be $\text{cov}(\tilde{y}_1, \tilde{y}_2) = \gamma / \sqrt{s_1 s_2} + \rho / \sqrt{h_1 h_2}$. The first component characterizes the client business risks

¹¹ The expected mean of $\tilde{\omega}_i$ is set equal to zero in the model, although some people may think of it as a surrogate for the auditor’s bias associated with the client managements’ bias in their earnings \bar{x}_i . Obviously, these biases (i.e., the expected means) will affect audit judgments — the distribution of \tilde{y}_i . As it will become clear in Section 3, these biases have no effect on the auditor’s posterior precision of the clients’ earnings—what we call audit quality. Of course, these biases do affect the auditor’s posterior expected means of the clients’ earnings. This may also be interpreted as the auditor’s role of “business advisory” (Beck and Wu 2006) — that is, the auditor helps the clients with their business decisions so as to increase the clients’ expected earnings. In the latter situation, the “mean-variance” tradeoff will become an important consideration.

¹² In reality, the clients’ earnings distributions and the auditors’ judgment errors will be correlated insofar as auditors choose their audit approaches and audit staff members in accordance with their clients’ characteristics (see Section 5 for details). For modeling convenience, we can transform correlated random variables into uncorrelated ones in order to reduce the expositional complexity.

¹³ Alternatively, one may think of (1) as audit technology. The precision and correlation parameters represent two composite elements measuring audit structure (Cushing and Loebbecke 1986, Chapter 4). Related measures are the “variability” in and “consensus” of auditor judgments (Morris and Nichols 1988) and auditor judgment consensus (Mock and Turner 1981).

by the precision parameters s_1 and s_2 as well as by the correlation parameter γ . The second component characterizes the audit “detection risks” by h_1 , h_2 , and ρ , depending on the accounting firm’s specific engagement factors such as audit staff assignments; audit policies, methodologies, and processes; and audit decision aids, etc.

As a means of further interpreting the basic model elements, we assume a pyramid-shaped audit-staffing configuration, wherein a single partner oversees two audit engagements, as depicted in Figure 1. Underneath the firm’s partner, a different manager is assigned to each engagement; and below each engagement manager are seniors and staff (who are not explicitly distinguished in the model). For simplicity, we assume that “ m ” people work exclusively on Engagement 1, “ n ” people work exclusively on Engagement 2, and “ k ” people work on both the engagements. These k personnel could include industry and technical specialists, but such expertise is not necessarily required. Thus, Engagements 1 and 2 collectively utilize $m + n + k$ employees (where m , n , and k are finite, nonnegative integers), and $k/(m + k + n)$ denotes the “staff overlap ratio.” *Ceteris paribus*, increasing this ratio should increase the correlation ρ between the auditors’ judgment errors.¹⁴ One simple example is the direct proportional case in which $\rho = ck/(m + k + n)$, where c is a nonnegative constant.

(Insert Figure 1 about here.)

Whether or not precision h_1 and h_2 are affected by k , however, depends on specifics. When the overlapping members have specialized in client industry or technical expertise, the precision is likely to increase with k . Alternatively, when the overlapping members are assigned

¹⁴ The staff overlap ratio is just one possible way to characterize the degree of uniformity in the audit structure, and it is intuitive and tractable. Alternatively, we may think of the client’s earnings as a vector of balance sheet valuation numbers (see details in Scott 1975, 103), so that each member of the engagement team will report on one of these random numbers. Although more descriptive, this alternative structure is unnecessarily complex since at the aggregate level, it is isomorphic to ours specified in (1).

to different audit teams on an “available basis” (see AOMAR Survey 2005), the precision could actually be sacrificed, as would be the case where the overlapping members have lower levels of competence than the other staff members on the audit engagement.

While the correlation between audit judgment errors and the correlation between client business risks help to link these engagements, the variance of each client’s business risks and the variance of each auditor’s judgment errors are equally important for characterizing each engagement. Accordingly, we refer to s_i / h_i as the *relative precision* of Engagement i ; and we assume that the relative precision of Engagement 1 is not larger than that of Engagement 2 (i.e., $s_1 / h_1 \leq s_2 / h_2$), without loss of generality.¹⁵ Equivalently, the “relative risk” of Engagement 1 is not smaller than that of Engagement 2: $\text{var}(\tilde{x}_1) / \text{var}(\tilde{\omega}_1) \geq \text{var}(\tilde{x}_2) / \text{var}(\tilde{\omega}_2)$. In addition, we assume that the earnings of each client and the representations of the two auditors are jointly normal random variables, that is

$$\begin{pmatrix} \tilde{x}_i \\ \tilde{y}_i \\ \tilde{y}_j \end{pmatrix} \sim N \left(\begin{pmatrix} \bar{x}_i \\ \bar{x}_i \\ \bar{x}_j \end{pmatrix}, \begin{pmatrix} 1/s_i & 1/s_i & \gamma/\sqrt{s_1 s_2} \\ 1/s_i & 1/s_i + 1/h_i & \gamma/\sqrt{s_1 s_2} + \rho/\sqrt{h_1 h_2} \\ \gamma/\sqrt{s_1 s_2} & \gamma/\sqrt{s_1 s_2} + \rho/\sqrt{h_1 h_2} & 1/s_j + 1/h_j \end{pmatrix} \right). \quad (2)$$

3. Audit quality and value of knowledge sharing

The manager/auditor of Engagement i will update the posterior beliefs about Client i ’s earnings distribution so that the posterior variance is $\text{var}(\tilde{x}_i | \tilde{y}_i = y_i) = 1/(s_i + h_i)$, for $i = 1, 2$. This posterior variance represents the auditor’s remaining uncertainty after taking into account all information available to the audit team (see other Bayesian models in Scott 1973; 1975; and

¹⁵ If the relationship between the two relative precisions were reversed, the two engagements could be relabeled so that the parameter relationship would hold. Nonetheless, the more intriguing issue is the relative precision construct. It represents the relative difficulty that the audit has in making judgments after controlling for the inherent variability in the client’s earning distribution. This characterization helps to distinguish variability present in the auditor’s judgment from the level of variability that is inherent in the client’s earnings itself.

Kinney 1975). The posterior variance has been used as a surrogate for audit risk (Simunic and Stein 1990). Hence, the precision (the reciprocal of the variance) of the posterior distribution provides a natural surrogate for audit quality (Beck and Wu 2006).

In our model, the accounting firm’s partner supervises both audit engagements and hence has access to information from both engagement teams (y_1 and y_2 of both clients’ earnings). Since the clients have economic relationships (γ is the correlation), and since the audit judgment errors corresponding to the managers’ assessments are also correlated (ρ is the correlation), observing both y_1 and y_2 will allow the partner of the firm to update her beliefs about each client’s earnings distribution. So, the partner’s posterior precision (for $i, j = 1, 2$, and $i \neq j$) is

$$q_i \equiv 1/\text{var}(\tilde{x}_i | \tilde{y}_1 = y_1, \tilde{y}_2 = y_2) = \frac{(s_1 + h_1)(s_2 + h_2) - (\sqrt{s_1 s_2} \rho + \sqrt{h_1 h_2} \gamma)^2}{s_j(1 - \rho^2) + h_j(1 - \gamma^2)}. \quad (3)$$

Recalling the assumption that $s_1/h_1 \leq s_2/h_2$, we obtain that $q_1/h_1 \leq q_2/h_2$ and $q_1/s_1 \geq q_2/s_2$.

The incremental information helps to make the partner’s posterior distribution potentially “tighter” than the two posterior distributions assessed “separately” by the engagement managers. Such an improvement in the posterior precision (or a reduction in the posterior variance) is due to the “pooling” of information about the clients’ business risks as well as about the engagement teams’ audit knowledge. Accordingly, we think of the “value” of knowledge dissemination from Engagement j to Engagement i as an increasing function of such incremental posterior precision (see Simunic 1984 and Wu 2006 for discussions of directional knowledge spillovers between audit and non-audit services). For the sake of expositional simplicity, we directly equate the value to the precision improvement:¹⁶

¹⁶ Many potential benefits can arise from sharing knowledge, e.g., audit effectiveness and efficiency, audit cost savings, etc. However, any monotonic transformation of the value function (5) will not change our results.

$$V_{ij} \equiv 1/\text{var}(\tilde{x}_i | \tilde{y}_1 = y_1, \tilde{y}_2 = y_2) - 1/\text{var}(\tilde{x}_i | \tilde{y}_i = y_i) \quad (4)$$

$$= \frac{s_j h_i (\rho - a_{ij} \gamma)^2}{s_j (1 - \rho^2) + h_j (1 - \gamma^2)}, \quad (5)$$

for $i, j = 1, 2$, and $i \neq j$; where $a_{ij} \equiv \sqrt{(s_i / h_i) / (s_j / h_j)}$ is the ratio between the standard relative precisions of Engagements i and j . Thus $a_{12} a_{21} = 1$ and $a_{12} \leq 1$. If the clients' earnings distributions were perfectly correlated ($\gamma = \pm 1$) and the audit-judgment errors were perfectly correlated ($\rho = 1$), then the two audit engagements would be indistinguishable from one another. Accordingly, we henceforth rule out these trivial cases by assuming that $\gamma^2 \rho \neq 1$.

An inspection of (5) indicates that, since s_j , h_i , and h_j are positive, V_{ij} will be positive unless $\rho = a_{ij} \gamma$. In particular, when the clients' earnings distributions are negatively correlated ($\gamma < 0$), the benefits from transferring knowledge across different audit engagements (as $\rho > 0$) and from diversifying client-business risks at the client-portfolio level (as $\gamma < 0$) are mutually reinforcing (i.e., $\rho - a_{ij} \gamma = \rho + a_{ij} |\gamma|$). Alternatively, when $\gamma > 0$, the benefit of knowledge sharing (as $\rho > 0$) will indeed counterbalance the firm's inability to diversify its clients' business risks (due to $\gamma > 0$). We now are able to state the following proposition.

PROPOSITION 1. *The value of disseminating knowledge from Engagement j to Engagement i is always positive unless $\gamma \geq 0$ and $\rho = a_{ij} \gamma$ (for $i, j = 1, 2$, and $i \neq j$).*

While knowledge dissemination generally improves audit quality, there are four special cases in which such improvement in audit quality will *not* occur. The first special case is when $\gamma = \rho = 0$ (as illustrated by the origin of Figure 2)—that is, the clients' business risks are uncorrelated with each other, as are the auditors' judgment errors. In this special case, the two

audit engagements are *de facto* “separate.” (Recall that parameter γ measures the strength of the clients’ economic relationships, and parameter ρ measures the degree of uniformity of audit judgments.) Such an environment could exist only when the clients’ business risks are independent of each other, and when the auditors share no staff members and use entirely different audit approaches as well.¹⁷ Under these rather extreme circumstances, it is intuitive that no knowledge sharing can take place between the engagements and, hence, $V_{12} = V_{21} = 0$.

(Insert Figure 2 about here.)

A second special case is when $\rho = \gamma > 0$ and $a_{12} = a_{21} = 1$ (as illustrated by the diagonal dotted line of Figure 2)—that is, the clients’ economic relationships are correlated to the same extent as are the auditors’ judgment errors, and the relative risks of the two engagements are equal. Thus, the correlation between the auditors’ judgment errors reduces the incremental informativeness of the second signal to the same extent that the clients’ business-risk correlation augments the dissemination of knowledge, so the two effects are precisely offsetting each other, and $V_{12} = V_{21} = 0$.

The third and fourth special cases occur when the relative risks of the two engagements are directly proportional to the relative magnitude of the two correlations (as illustrated by the two off-diagonal lines of Figure 2). Case 3 occurs when $\rho/\gamma = a_{12}$ (the dashed off-diagonal line), while Case 4 occurs when $\rho/\gamma = a_{21}$ (the dotted off-diagonal line). Substituting the parameter relation in Case 3 into (5) yields $V_{12} = 0$ and $V_{21} > 0$ (note, $a_{21}\gamma - \rho > \gamma - \rho > 0$): Knowledge only transfers from Engagement 1 to Engagement 2. Conversely, substituting the

¹⁷ In turn, this would require that the auditors be given complete latitude in planning and executing their audits and that they structure highly diverse audit approaches so that their judgment errors are independent of each other.

relation in Case 4 into (5) yields $V_{21} = 0$ and $V_{12} > 0$ (note, $\rho - a_{12}\gamma > \rho - \gamma > 0$): Knowledge only disseminates from Engagement 2 to Engagement 1.

Except for these aforementioned special cases, however, Proposition 1 indicates that information-sharing benefits will accrue to the accounting firm. For example, when the earnings distributions of the clients are negatively associated, there is always a positive benefit from knowledge sharing. Hence, the magnitude of the knowledge-sharing benefit and hence audit quality will depend on the prior precisions of information as well as on the correlation between the auditors' judgment errors and the correlation between the clients' business risks. Note, from (4) and (3), that audit quality is the sum of the manager's posterior precision and the knowledge-transfer value: $q_i = (s_i + h_i) + V_{ij}$. Therefore, the following lemma is immediate.

LEMMA 1. *Audit quality and the value of knowledge sharing are related to each other in terms of the prior precisions of information, the degree of uniformity in audit judgments, and the correlation of the clients' business risks. Specifically (for $i, j = 1, 2$ and $i \neq j$)*

$$\frac{\partial q_i}{\partial \ell} = \frac{\partial V_{ij}}{\partial \ell}, \quad \text{for } \ell \in \{s_j, h_j, \rho, \gamma\}; \text{ and} \quad (6)$$

$$\frac{\partial q_i}{\partial \ell} = \frac{\partial V_{ij}}{\partial \ell} + 1, \quad \text{for } \ell \in \{s_i, h_i\}. \quad (7)$$

Equation (6) indicates that perturbations in the model parameters have the same impact on audit quality as they have on the value of knowledge sharing. These model parameters include the prior precision of Client j 's earnings distribution, the prior precision of Engagement j 's audit judgments, the correlation between audit judgment errors, and the correlation between the clients' business risks. Thus, their effects on audit quality and on the value of knowledge sharing are identical.

On the other hand, Equation (7) indicates that the impact that the prior precisions of Client i 's earnings distribution and the Engagement i 's auditor's judgments have on the audit quality of Engagement i is equal to one plus the impact of these prior precisions on the value of disseminating knowledge from Engagement j to Engagement i . The economic intuition is simple. Even if an increase in the engagement managers' prior precisions does not affect the value of knowledge sharing, it still helps to enhance the posterior precision, because the posterior distribution is simply being updated from the prior distribution (Bhattacharjee, Maletta and Moreno 2005).

4. Comparative static analyses

Given that the knowledge-sharing value and audit quality are closely related, we henceforth focus on the comparative static analyses on audit quality. To sharpen the economic intuition, we first consider a benchmark case in which the relative precisions of the audit judgments for the two engagements are equal in Subsection 4.1 — what we call symmetric audit engagements. Then, we extend the analysis to the general case in which the relative precisions of the audit judgments for the two engagements are unequal in Subsection 4.2 — what we call asymmetric audit engagements.

4.1. Benchmark: symmetric audit engagements

The simplest case involves two audit engagements that are symmetric in terms of client business risks and auditor judgment errors. Specifically, the ratios of client-business risk over audit-judgment error are equal: $\text{var}(\tilde{x}_1)/\text{var}(\tilde{\omega}_1) = \text{var}(\tilde{x}_2)/\text{var}(\tilde{\omega}_2)$ (i.e., $s_1/h_1 = s_2/h_2$, or $a_{12} = a_{21} = 1$). Thus, rewriting (3) yields,

$$q_i = h_i \frac{(s_i/h_i + 1)(s_j/h_j + 1) - [\rho\sqrt{s_i s_j / (h_i h_j)} + \gamma]^2}{(s_j/h_j)(1 - \rho^2) + (1 - \gamma^2)} = \frac{(s_i + h_i)^2 - (s_i \rho + h_i \gamma)^2}{s_i(1 - \rho^2) + h_i(1 - \gamma^2)}. \quad (8)$$

Denote by s/h the equal ratio of the relative risks ($s_1/h_1 = s_2/h_2$). Taking the partial derivatives of (8) regarding s and h yields,

$$\frac{\partial q_i}{\partial s} = 1 + \frac{h^2(1 - \gamma^2)(\rho - \gamma)^2}{[s(1 - \rho^2) + h(1 - \gamma^2)]^2} > 0, \quad \text{and} \quad (9)$$

$$\frac{\partial q_i}{\partial h} = 1 + \frac{s^2(1 - \rho^2)(\rho - \gamma)^2}{[s(1 - \rho^2) + h(1 - \gamma^2)]^2} > 0. \quad (10)$$

PROPOSITION 2. *Assume that the audit engagements have equal level of relative risks. Then, audit quality will be increasing with the prior precisions of the client's earnings distribution and the auditors' judgments.*

As expected intuitively, these relations of audit quality with the prior precisions (s and h) of the client's earnings distribution and with the auditor's judgment errors (see (9) and (10)) are always positive. These relations remain intact even if $\rho = \gamma$ in addition to $a_{12} = a_{21} = 1$ —the second special case following Proposition 1—whereas the value of knowledge sharing equals zero. This implies that knowledge sharing between the two symmetric engagements is simply superfluous. The key condition is $\rho = \gamma$, which means that the clients' business risks are correlated to the “same extent” that the auditors' judgment errors are correlated. Substituting $\rho = \gamma$ into (8) yields $q_i = s_i + h_i = 1/\text{var}(\tilde{x}_i | \tilde{y}_i = y_i)$: The audit quality (the posterior precision conditional on both y_1 and y_2) equals the posterior precision conditional only on y_i .

Taking the partial derivatives of q_i in (8) with respect to ρ and γ yields,

$$\text{sign}\{\partial q_i / \partial \rho\} = \text{sign}\{\rho - \gamma\}, \quad \text{and} \quad (11)$$

$$\text{sign}\{\partial q_i / \partial \gamma\} = \text{sign}\{\gamma - \rho\}. \quad (12)$$

Hence, the following proposition is immediate.

PROPOSITION 3. *Assume that the audit engagements have equal level of relative risks. Then, audit quality q_i varies with ρ and γ as follows ($i=1, 2$):*

$$(a.) \quad \partial q_i / \partial \rho > 0 \text{ and } \partial q_i / \partial \gamma < 0, \text{ if } \rho > \gamma, \text{ and } \gamma \in [-1, 1]; \quad (13)$$

$$(b.) \quad \partial q_i / \partial \rho = 0 \text{ and } \partial q_i / \partial \gamma = 0, \text{ if } \rho = \gamma, \text{ and } \gamma \in [0, 1]; \quad (14)$$

$$(c.) \quad \partial q_i / \partial \rho < 0 \text{ and } \partial q_i / \partial \gamma > 0, \text{ if } \rho < \gamma, \text{ and } \gamma \in (0, 1]. \quad (15)$$

The most noteworthy implication of Proposition 3 is that audit quality may be either increasing or decreasing in the correlation between the auditors' judgment errors (ρ) as well as in the correlation between the clients' business risks (γ). Figure 3 illustrates the results visually. In Region I of Figure 3, the clients' earnings are negatively correlated. With a negative correlation between the clients' earnings distributions ($\gamma < 0$, as illustrated by Region I of Figure 3), it is apparent that audit quality will be increasing with the correlation ρ between the auditors' judgment errors (which is assumed to be non-negative in the model). Thus, knowledge-sharing strategies (such as overlapping more auditor specialists across engagements) will improve audit quality. Additionally, as the clients' business-risk correlation becomes more highly negative, the knowledge-sharing benefit will be further enhanced — that is, both the benefits of risk diversification and knowledge dissemination are mutually enforcing. This result is consistent with the finding of Proposition 1 that combining the information from both audit engagements has a strictly positive value of sharing knowledge when $\gamma < 0$, and in that special case, the value of knowledge sharing increases monotonically with the magnitude of ρ and γ .

(Insert Figure 3 about here.)

Alternatively, the clients' business risks could be “positively” correlated (as illustrated in Regions II and III of Figure 3 which are separated by the 45-degree line along that $\gamma = \rho$). In

this special case, the clients' business risk at the client-portfolio level will be *de facto* elevated. Therefore, the effect of “risk diversification” is diminishing (more precisely, lack of risk diversification), and is offsetting the benefit from knowledge sharing. In particular, when $\rho = \gamma$, these effects of risk diversification and knowledge dissemination on audit quality are perfectly counterbalanced; thus their net effect is zero — consistent with the finding of Proposition 1 that there exist no audit-quality benefits from sharing knowledge.

In Region II where the correlation between the auditors' judgment errors is larger than the correlation between the clients' business risks ($\rho > \gamma$), the impact of knowledge sharing on audit quality remains positive, despite the fact that increasing the strength of this positive correlation between the clients' business risks reduces audit quality. That is, the knowledge-sharing benefit predominates. In Region III where $\rho < \gamma$, the above results are reversed with the risk-diversification effect predominating.

4.2. Asymmetric audit engagements

The comparative static analyses for the benchmark case (wherein both audit engagements have equal relative precisions and relative risks, $a_{12} = a_{21} = 1$) are quite intuitive. Nonetheless, the main results rely on a key separating-boundary condition that the clients' economic relationships are correlated to the same extent that the auditors' judgment errors are correlated: $\rho = \gamma$. To see the full ramifications of judgment uniformity on audit quality, we now consider the general, more realistic case in which the two audit engagements have “unequal” relative precisions and relative risks. Taking the partial derivatives of V_{ij} specified in (5) with respect to ρ and γ yields (see Lemma 1)

$$\frac{\partial q_i}{\partial \rho} = \frac{\partial V_{ij}}{\partial \rho} = \frac{2s_j h_i}{[s_j(1-\rho^2) + h_j(1-\gamma^2)]^2} (\rho - a_{ij}\gamma) [s_j(1 - a_{ij}\gamma\rho) + h_j(1 - \gamma^2)], \quad (16)$$

$$\frac{\partial q_i}{\partial \gamma} = \frac{\partial V_{ij}}{\partial \gamma} = \frac{-2s_j h_i}{[s_j(1-\rho^2) + h_j(1-\gamma^2)]^2} (\rho - a_{ij}\gamma) [a_{ij}s_j(1-\rho^2) + h_j(a_{ij} - \gamma\rho)]. \quad (17)$$

Therefore, here is the next proposition (see the proof in the Appendix).

PROPOSITION 4. *Both audit quality and the knowledge-sharing value vary with the correlation between the auditors' judgment errors and the correlation between the clients' business risks. Specifically, if $\gamma \leq 0$, then (for $i, j = 1, 2$ and $i \neq j$)*

$$\partial q_i / \partial \rho = \partial V_{ij} / \partial \rho > 0 \text{ and } \partial q_i / \partial \gamma = \partial V_{ij} / \partial \gamma < 0. \quad (18)$$

Alternatively, if $\gamma > 0$, then

(a) *There exist boundary conditions $\rho^\circ \equiv a_{12}\gamma$ and $\rho^* \equiv \sqrt{\frac{1+h_2/s_2}{1+h_1/s_1}}$, so that q_1*

and V_{12} vary with ρ and γ as follows:

$$\frac{\partial q_1}{\partial \rho} = \frac{\partial V_{12}}{\partial \rho} = \begin{cases} > 0, & \text{if } \rho > \rho^\circ, \\ < 0, & \text{if } \rho < \rho^\circ; \end{cases} \text{ and} \quad (19)$$

$$\frac{\partial q_1}{\partial \gamma} = \frac{\partial V_{12}}{\partial \gamma} = \begin{cases} < 0, & \text{if } \rho^\circ < \rho \leq \rho^*, \\ > 0, & \text{if } \rho < \rho^\circ. \end{cases} \quad (20)$$

(b) *There exist boundary conditions $\rho^\bullet \equiv a_{21}\gamma$ and $\gamma^* \equiv \sqrt{\frac{1+s_1/h_1}{1+s_2/h_2}}$, so that q_2*

and V_{21} vary with ρ and γ as follows:

$$\frac{\partial q_2}{\partial \rho} = \frac{\partial V_{21}}{\partial \rho} = \begin{cases} > 0, & \text{if } \rho > \rho^\bullet, \\ < 0, & \text{if } \rho < \rho^\bullet \text{ and } \gamma \leq \gamma^*; \end{cases} \text{ and} \quad (21)$$

$$\frac{\partial q_2}{\partial \gamma} = \frac{\partial V_{21}}{\partial \gamma} = \begin{cases} < 0, & \text{if } \rho > \rho^\bullet, \\ > 0, & \text{if } \rho < \rho^\bullet. \end{cases} \quad (22)$$

Although the conditions identified in Proposition 4 are more complicated than those in Proposition 3, the essential intuition remains intact. Increasing the degree of uniformity among audit judgments is always mutually beneficial for both the audit engagements when $\gamma < 0$. The economic intuition is simple: When the clients' business risks are "negatively" correlated, both the benefits from risk diversification and knowledge dissemination are mutually reinforcing.

Alternatively, when the clients' business risks are "positively" correlated, the lack of risk diversification at the client portfolio level will offset the effect of knowledge dissemination. Hence, their net impact on audit quality will be parameter specific. We find once again that audit quality and the knowledge-sharing benefit do not have a "monotone" relation with either ρ or γ over the entire range of these parameters. Depending on how ρ and γ relate to each other (see details in Figure 4), the relation between the audit quality and the knowledge-sharing benefit may be either increasing or decreasing.

(Insert Figure 4 about here.)

The key implication of Propositions 3 and 4 is that both audit quality and the knowledge-sharing benefit depend *jointly* on the clients' "clustering" characteristics (for example, client industry, client affiliation groupings, and market conditions) and the auditors' judgment-error correlation. Therefore, public accounting firms can no longer view the risk of one audit engagement as being distinct or isolated from that of other engagements. This finding, of course, supports Simunic and Stein's (1990) main thesis that the auditor should take a "portfolio" perspective when evaluating his or her clients' business risks. A distinguishing implication of our analysis, however, is that the benefit from sharing knowledge must be evaluated at the firm level in the context of the clients' correlated business risks. When clients are clustered, their business risks are likely to be significantly correlated. To the extent that affiliated companies (either in a consolidated group or in the same industry) are audited by the identical audit process or by the same individual, there also could be a fairly high degree of correlated audit-judgment errors. The magnitude of auditor judgment-error correlation will be further exacerbated by the extent to which there is a positive correlation between the clients' business risks. Consequently,

public accounting firms ought to recognize the “interaction” between the clients’ business clustering and the auditors’ judgment errors.

Up to this point, we have focused our analysis on the impact that the correlation between clients’ business risks and the correlation between the auditors’ judgment errors have on audit quality. However, audit quality will also be influenced by the precisions of the clients’ business risks and the audit approach. Accordingly, we now turn to the comparative static analyses of audit quality with respect to s_1 , s_2 , h_1 , and h_2 .

PROPOSITION 5. *Audit quality varies with the prior precisions of each client’s earnings distribution and the auditor’s judgments. Specifically, for $i, j = 1, 2$ and $i \neq j$*

(a) *If $\gamma < 0$, then (for all $\rho \geq 0$)*

$$\partial q_i / \partial h_i > 0, \text{ and } \partial q_i / \partial s_i > 0. \quad (23)$$

(b) *If $\rho = 0$, then (for all $\gamma \in [-1, 1]$)*

$$\partial q_i / \partial h_i > 0, \text{ and } \partial q_i / \partial s_i > 0; \quad (24)$$

$$\partial q_i / \partial h_j > 0, \text{ and } \partial q_i / \partial s_j < 0. \quad (25)$$

(c) *If $\rho > 0$ and $\gamma = 0$, then*

$$\partial q_i / \partial h_i > 0, \text{ and } \partial q_i / \partial s_i > 0; \quad (26)$$

$$\partial q_i / \partial h_j < 0, \text{ and } \partial q_i / \partial s_j > 0. \quad (27)$$

(d) *If $\rho > 0$ and $\gamma > 0$, then*

$$\partial q_1 / \partial h_1 > 0, \text{ and } \partial q_2 / \partial s_2 > 0; \quad (28)$$

$$\partial q_2 / \partial h_2 > 0 \text{ if } \gamma \leq \gamma^*, \text{ and } \partial q_1 / \partial s_1 > 0 \text{ if } \rho \leq \rho^*; \quad (29)$$

$$\text{sign} \left\{ \frac{\partial q_i}{\partial h_j} \right\} = -\text{sign} \{ \rho - a_{ij} \gamma \}, \text{ and } \text{sign} \left\{ \frac{\partial q_i}{\partial s_j} \right\} = \text{sign} \{ \rho - a_{ij} \gamma \}. \quad (30)$$

Consistent with the benchmark results of Proposition 2, the audit quality of each engagement is increasing with the precision of clients’ earnings distributions and with the precision of the audit judgment errors at the engagement level. Except for the two knife-edge

cases specified by the complements of (29)—that is, $\rho > \rho^*$ and $\gamma > 0$; and $\gamma > \gamma^*$ (see Figure 4)—both $\partial q_i / \partial h_i$ and $\partial q_i / \partial s_i$ remain to be positive, irrespective of whether audit engagements are symmetric. However, audit quality is not monotonically increasing in the engagement precision parameters when the two engagements initially do not have equal relative precisions. Even when $\gamma < 0$, the signs of $\partial q_i / \partial h_j$ and $\partial q_i / \partial s_j$ are undetermined. In general, depending on how ρ and γ relate to each other, an increase in the prior precision of a client’s earnings distribution or the precision of audit approach on one engagement may or may not enhance the audit quality of the other engagement—see (25), (27), and (30).

5. Impact of audit-staffing policies on audit quality

In the previous section, our comparative static analyses have showed that audit quality varies neither monotonically with the correlation nor monotonically with the precisions of the audit judgments. Because public accounting firms employ their own audit staffing policies and different levels of audit structure (Cushing and Loebbecke 1986; Kinney 1986), both the correlation and the precision parameters of the auditor’s judgment in the model can vary either “separately” or “jointly” as depicted in Figure 4.1 of Cushing and Loebbecke (1986). For example, overlapping staff members among several audit engagements is a common approach for auditors to utilize and leverage their industry specialists and experts—to which Simunic and Stein (1987) refer as one way of differentiating products in auditing. In general, the overlapping members of different audit teams are likely to “jointly” affect the correlation as well as the precision (ρ and h_i) of the partner’s audit judgments, so that their overall impact on audit

quality will be complex.¹⁸ Intuitively, we would expect that increasing the number of overlapping staff members will increase the correlation among audit judgment errors. The precision of audit judgments, however, may be either increased or decreased, depending on whether these overlapping members do possess *de facto* incremental expertise. To the extent that the overlapping staff members possess specialized knowledge and expertise, audit quality can be enhanced. However, if they lack incremental expertise, the overall precision will not be improved and, in some cases, could actually decrease.

As depicted by Figure 1, parameter k represents the number of audit staff members who overlap the two engagements. Therefore, “totally” differentiating (3) with respect to k yields (for $i, j = 1, 2$ and $i \neq j$)

$$\frac{dq_i}{dk} = \frac{\partial q_i}{\partial \rho} \frac{\partial \rho}{\partial k} + \frac{\partial q_i}{\partial h_i} \frac{\partial h_i}{\partial k} + \frac{\partial q_i}{\partial h_j} \frac{\partial h_j}{\partial k}. \quad (31)$$

We now can delineate more precisely conditions under which audit staff overlap helps to enhance audit quality. For concreteness, we will illustrate these conditions through the following specific examples in order to explain intuitively the overall impact of k on audit quality,

$$\rho = \frac{ck}{m+k+n}, \quad h_1 = \frac{c+k}{m+k}, \quad \text{and} \quad h_2 = \frac{c+k}{n+k}. \quad (32)$$

Hence,

$$\frac{\partial \rho}{\partial k} = \frac{c(m+n)}{(m+k+n)^2} = \begin{cases} > 0, & \text{if } c > 0, \\ = 0, & \text{if } c = 0; \end{cases} \quad (33)$$

¹⁸ In practice, staff overlap can occur in a variety of contexts. One example of audit staff overlap would involve the assignment of the same auditors to clients that are affiliated (parents and their subsidiaries) or to clients that are supply-chain partners. A second example of audit staff overlap involves the assignment of “technical experts” in areas such as information systems, statistical sampling, or financial instruments to work on multiple engagements. Finally, a third example of audit staff overlap could involve assigning auditors to clients in the same industry as a means of developing “industry specialization and expertise.”

$$\frac{\partial h_1}{\partial k} = \frac{m-c}{(m+k)^2} = \begin{cases} > 0, & \text{if } m > c, \\ < 0, & \text{if } m < c; \end{cases} \text{ and} \quad (34)$$

$$\frac{\partial h_2}{\partial k} = \frac{n-c}{(n+k)^2} = \begin{cases} > 0, & \text{if } n > c, \\ < 0, & \text{if } n < c. \end{cases} \quad (35)$$

Loosely speaking, the parameter c can be interpreted as the minimum number of staff required to perform the engagements. Hence, the audit-judgment-error correlation is increasing with k if and only if c is greater than zero, as detailed in (33). In addition, the prior precisions of audit judgments are increasing with k if, and only if, the engagements meet the minimum staffing requirement ($m > c$ and $n > c$), as detailed in (34) and (35). Alternatively, in the special case where $m = n = c$, the precisions of the audit judgments are a constant and, thus, will not be affected by varying the staff overlap. Hence, the overall impact of k on audit quality is equivalent to the partial impact of ρ on audit quality: $\text{sign}\{dq_i / dk\} = \text{sign}\{\partial q_i / \partial \rho\}$. Substituting these comparative static relations of Propositions 4 and 5 into (31) yields the relations in Table 1.

(Insert Table 1 about here.)

Consequently, we immediately obtain the following corollaries.

COROLLARY 1. *If $c = 0$, then $dq_i / dk > 0$ (for $i = 1, 2$).*

COROLLARY 2. *If $m > c > n$, then $dq_1 / dk > 0$, provided that $\rho > \rho^\circ \equiv a_{12}\gamma \geq 0$.*

Alternatively, if $n > c > m$, then $dq_2 / dk > 0$, provided that $\rho > \rho^\bullet \equiv a_{21}\gamma \geq 0$.

COROLLARY 3. *If $m < c$ and $n < c$, then $dq_1 / dk < 0$, provided that $0 < \rho < \rho^\circ \equiv a_{12}\gamma$; and $dq_2 / dk < 0$, provided that $0 < \rho < \rho^\bullet \equiv a_{21}\gamma < a_{21}\gamma^*$.*

The result of Corollary 1 is intuitive. When there is no minimum audit-staffing requirement ($c = 0$, see the second row of Table 1), an increase in audit staff overlap will always enhance audit quality, although it has no impact on the uniformity of audit approaches applied to

the audit engagements (note, $\rho = 0$ if $c = 0$). The reason is that under these conditions, audit staff overlap will always help to increase the precision of auditor judgments.

In Corollary 2, we identify two sets of conditions under which audit staff overlap helps to enhance audit quality. One set of conditions (see the third row of Table 1) is when the clients' business risks are "independent," the audit judgment errors are positively correlated, and one audit engagement (but not the other) meets the minimum staff requirement. The other set of conditions (see the first and third cases on the fourth row of Table 1) is when the clients' business risks are positively correlated, the auditors' judgment errors are also highly correlated, and one audit engagement (but not the other) meets the minimum staff requirement. Under each set of conditions, an increase in audit staff overlap will consistently enhance the audit quality of the engagement (when that engagement meets the minimum staff requirement, of course).

In Corollary 3, we assume that neither of the two audit engagements meets the minimum audit-staffing requirement (that is, $m < c$ and $n < c$), as illustrated by the second and fourth cases on the fourth row of Table 1. Hence, an increase in audit staff overlap will decrease audit quality, provided that the ratio of audit-judgment-error and client-business-risk correlations is lower than the relative precision ratio of the two engagements, and the client-business-risk correlation is not excessively high.

6. Concluding remarks and implications

We have examined the impact of audit-judgment-enhancement strategies on audit quality in a stylized model where an accounting firm provides audit services for two clients having correlated business risks. Increasing the degree of uniformity in auditing practice generally facilitates knowledge dissemination among audit engagements and, thus, enhances audit quality. When clients have correlated business risks, however, strategies used to acquire, disseminate,

and leverage expertise across multiple audit engagements can either attenuate or exacerbate the firm's risk exposure as a whole (a portfolio of clients). Building on the main thesis of Simunic and Stein (1990) that clients' business risks should be evaluated at the portfolio level, we show that auditors' judgment errors should also be evaluated and managed at the firm level, rather than at the engagement level. Hence, the covariance between clients' business risks and the covariance between auditors' judgment errors do interact with each other and, thus, jointly affect the accounting firm's overall risk exposure as a whole. When the precision improvement is small, therefore, the effect due to the induced correlation in judgment errors (see over-confidence in judgment by Gibbins 1982) could predominate and ultimately result in lower audit quality.

Our result provides new insight into audit staffing policies. When clients' business risks are negatively correlated, assigning the same audit team (versus separate audit teams) to these clients will enhance (versus reduce) audit quality, because the same audit team is more likely to reap the benefits from diversifying clients' business risks at the portfolio level than would separate audit teams. Conversely, when clients coordinate or share a significant amount of business activities (such as would be the case when they are supply-chain partners, or are affiliated companies), their business risks are likely to be highly, positively correlated. Thus, assigning the same audit team (versus two "separate" audit teams) to different audit clients will reduce (versus enhance) audit quality, because the same audit team is more likely to exacerbate judgment-error correlation than would separate audit teams.

This insight helps to explain the diversity in auditing practices of public accounting firms. Some firms allow partners and managers to "preference" staff members so that audit teams tend to remain intact as they move across a sequence of audit engagements, while others appear to rotate staff members in different industries in order to expose staff members to various groups of

clients. Another, related, auditing staffing policy employed by one Big 4 firm is to send the same audit team to audit all affiliated companies wherever possible; resulting in highly correlated judgment errors. In contrast, at another Big 4 firm, audits of subsidiaries are performed by offices in geographical proximity, rather than by the audit team performing the audit of the parent company. On the surface, these staffing policies seem to be in direct conflict. However, their survival over the years can be explained by the trade-off between the correlation of and the variability in audit judgment errors (conditional on the direction and magnitude of the clients' business-risk correlation).

Appendix: Proofs

Proof of Proposition 4:

Note that $\rho \geq 0$. Thus, if $\gamma \leq 0$, then it is obvious that (18) follows from (16) and (17).

Alternatively, if $\gamma > 0$, then assumption $a_{12} \leq 1$ implies $1 - a_{12}\gamma\rho \geq 0$, and assumption $a_{21} \geq 1$ implies $a_{21} - \gamma\rho \geq 0$. Thus, substituting them into (16) and (17), we get the following from (6)

$$\text{sign}\{\partial q_1 / \partial \rho\} = \text{sign}\{\partial V_{12} / \partial \rho\} = \text{sign}\{\rho - \rho^\circ\}, \text{ and} \quad (\text{A.1})$$

$$\text{sign}\{\partial q_2 / \partial \gamma\} = \text{sign}\{\partial V_{21} / \partial \gamma\} = -\text{sign}\{\rho - \rho^\bullet\}, \quad (\text{A.2})$$

which prove (19) and (22), respectively.

Now, we turn to the proof of (20). First, if $\rho > \rho^\circ$, then (6) and (17) imply,

$$\text{sign}\{\partial q_1 / \partial \gamma\} = \text{sign}\{\partial V_{12} / \partial \gamma\} = -\text{sign}\{s_2(1 - \rho^2) + h_2(1 - a_{21}\gamma\rho)\}. \quad (\text{A.3})$$

Note that the following inequalities are equivalent (we use “if, and only if,” or “iff” in short, to represent equivalence): $\rho > \rho^\circ \equiv a_{12}\gamma$ iff $a_{21}\rho > a_{21}a_{12}\gamma = \gamma$ iff $(a_{21}\rho)^2 > a_{21}\rho\gamma$ iff

$$1 - a_{21}\rho\gamma > 1 - (a_{21}\rho)^2 \text{ iff}$$

$$s_2(1 - \rho^2) + h_2(1 - a_{21}\gamma\rho) > s_2(1 - \rho^2) + h_2[1 - (a_{21}\rho)^2]. \quad (\text{A.4})$$

In addition, $\rho \leq \rho^*$ iff $\rho^2 \leq (1 + h_2/s_2)/(1 + h_1/s_1)$ iff $1 + h_2/s_2 - \rho^2(1 + h_1/s_1) \geq 0$ iff

$$s_2(1 - \rho^2) + h_2[1 - (a_{21}\rho)^2] \geq 0. \quad (\text{A.5})$$

Therefore, if $\rho^\circ < \rho \leq \rho^*$, then from (A.5), (A.4) and (A.3), we get that $\partial q_1 / \partial \gamma = \partial V_{12} / \partial \gamma < 0$, which is the first part of (20).

Second, if $\rho < \rho^\circ$, then (6) and (17) imply

$$\text{sign}\{\partial q_1 / \partial \gamma\} = \text{sign}\{\partial V_{12} / \partial \gamma\} = \text{sign}\{s_2(1 - \rho^2) + h_2(1 - a_{21}\gamma\rho)\}. \quad (\text{A.6})$$

Note, $\rho < \rho^\circ \equiv a_{12}\gamma$ iff $a_{21}\rho < a_{21}a_{12}\gamma = \gamma$ iff $a_{21}\rho\gamma < \gamma^2$ iff $1 - a_{21}\gamma\rho > 1 - \gamma^2$ iff

$$s_2(1 - \rho^2) + h_2(1 - a_{21}\gamma\rho) > s_2(1 - \rho^2) + h_2(1 - \gamma^2). \quad (\text{A.7})$$

Since $s_2(1 - \rho^2) + h_2(1 - \gamma^2) > 0$, (A.7) and (A.6) imply that $\partial q_1 / \partial \gamma = \partial V_{12} / \partial \gamma > 0$, which is the second part of (20).

Finally, we turn to the proof of (21). First, if $\rho > \rho^\bullet$, then (6) and (16) imply

$$\text{sign}\{\partial q_2 / \partial \rho\} = \text{sign}\{\partial V_{21} / \partial \rho\} = \text{sign}\{s_1(1 - a_{21}\gamma\rho) + h_1(1 - \gamma^2)\}. \quad (\text{A.8})$$

Note, $\rho > \rho^\bullet \equiv a_{21}\gamma$ iff $\rho^2 > a_{21}\gamma\rho$ iff $1 - a_{21}\gamma\rho > 1 - \rho^2$ iff

$$s_1(1 - a_{21}\gamma\rho) + h_1(1 - \gamma^2) > s_1(1 - \rho^2) + h_1(1 - \gamma^2). \quad (\text{A.9})$$

Since $s_1(1 - \rho^2) + h_1(1 - \gamma^2) > 0$, (A.9) and (A.8) imply that $\partial q_2 / \partial \rho = \partial V_{21} / \partial \rho > 0$, which is the first part of (21).

Second, if $\rho < \rho^\bullet$, then (6) and (16) imply,

$$\text{sign}\{\partial q_2 / \partial \rho\} = \text{sign}\{\partial V_{21} / \partial \rho\} = -\text{sign}\{s_1(1 - a_{21}\gamma\rho) + h_1(1 - \gamma^2)\}. \quad (\text{A.10})$$

Note, $\rho < \rho^\bullet \equiv a_{21}\gamma$ iff $(a_{21}\gamma)^2 > a_{21}\gamma\rho$ iff $1 - a_{21}\gamma\rho > 1 - (a_{21}\gamma)^2$ iff

$$s_1(1 - a_{21}\gamma\rho) + h_1(1 - \gamma^2) > s_1[1 - (a_{21}\gamma)^2] + h_1(1 - \gamma^2). \quad (\text{A.11})$$

In addition, $\gamma \leq \gamma^*$ iff $\gamma^2 \leq (1 + s_1 / h_1) / (1 + s_2 / h_2)$ iff $1 + s_1 / h_1 - \gamma^2(1 + s_2 / h_2) \geq 0$ iff

$$s_1[1 - (a_{21}\gamma)^2] + h_1(1 - \gamma^2) \geq 0. \quad (\text{A.12})$$

If $\rho < \rho^\bullet$ and $\gamma \leq \gamma^*$, from (A.12), (A.11) and (A.10), we get that $\partial q_2 / \partial \rho = \partial V_{21} / \partial \rho > 0$,

which is the second part of (21). ***Q.E.D.***

Proof of Proposition 5:

From (3), we take partial derivatives as follows (for $i, j = 1, 2$, and $i \neq j$):

$$\frac{\partial q_i}{\partial h_i} = \frac{s_j(1 - a_{ij}\gamma\rho) + h_j(1 - \gamma^2)}{s_j(1 - \rho^2) + h_j(1 - \gamma^2)}, \quad (\text{A.13})$$

$$\frac{\partial q_i}{\partial s_i} = \frac{s_j(1 - \rho^2) + h_j(1 - a_{ji}\gamma\rho)}{s_j(1 - \rho^2) + h_j(1 - \gamma^2)}; \text{ and} \quad (\text{A.14})$$

$$\frac{\partial q_i}{\partial h_j} = -(\rho - a_{ij}\gamma) \frac{s_j \sqrt{h_i/h_j} [\sqrt{s_1 s_2} (1 - \rho^2)\gamma + \sqrt{h_1 h_2} (1 - \gamma^2)\rho]}{[s_j(1 - \rho^2) + h_j(1 - \gamma^2)]^2}, \quad (\text{A.15})$$

$$\frac{\partial q_i}{\partial s_j} = (\rho - a_{ij}\gamma) \frac{\sqrt{h_1 h_2} [\sqrt{s_1 s_2} (1 - \rho^2)\gamma + \sqrt{h_1 h_2} (1 - \gamma^2)\rho]}{[s_j(1 - \rho^2) + h_j(1 - \gamma^2)]^2}. \quad (\text{A.16})$$

(a) Note that $\rho \geq 0$. Hence, inspecting (A.13) and (A.14) when $\gamma < 0$, we get (23).

(b) Substituting $\rho = 0$ into (A.13)-(A.16) yields (24).

(c) Substituting $\rho > 0$ and $\gamma = 0$ into (A.13)-(A.16) yields (25).

(d) Finally, we now consider the case in which $\rho > 0$ and $\gamma > 0$. First, noting that $a_{12} \leq 1$, we get (26).

Second, if $\rho^\circ < \rho \leq \rho^*$, (A.4) and (A.5) imply that $s_2(1 - \rho^2) + h_2(1 - a_{21}\gamma\rho) > 0$. In addition, if $\rho \leq \rho^\circ$, (A.7) implies that $s_2(1 - \rho^2) + h_2(1 - a_{21}\gamma\rho) > 0$. Thus, this proves (27).

Third, if $\rho < \rho^\bullet$ and $\gamma \leq \gamma^*$, (A.11) and (A.12) imply that $s_1(1 - a_{21}\gamma\rho) + h_1(1 - \gamma^2) > 0$. In addition, if $\rho \geq \rho^\bullet$, (A.9) implies that $s_1(1 - a_{21}\gamma\rho) + h_1(1 - \gamma^2) > 0$. Thus, this proves (28).

Finally, inspecting (A.15) and (A.16) when $\rho > 0$ and $\gamma > 0$, we get (29) and (30).

Q.E.D.

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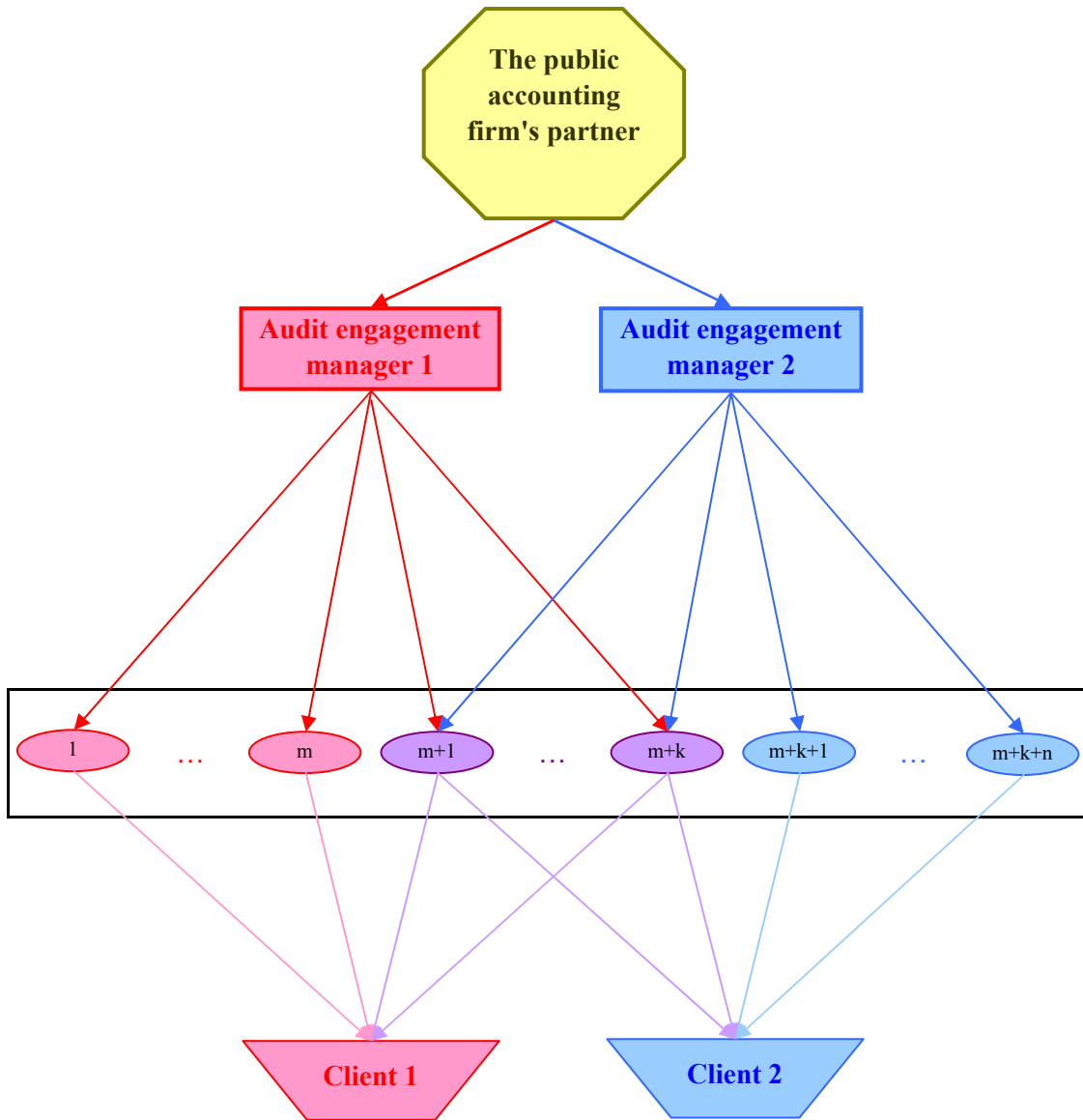
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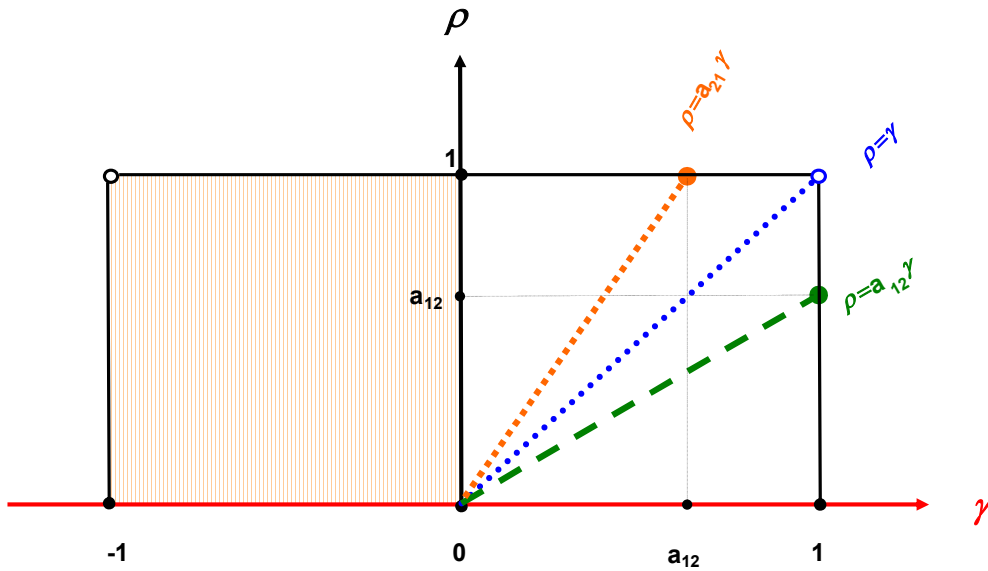
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Figure 1: The pyramid-shape structure of the public accounting firm



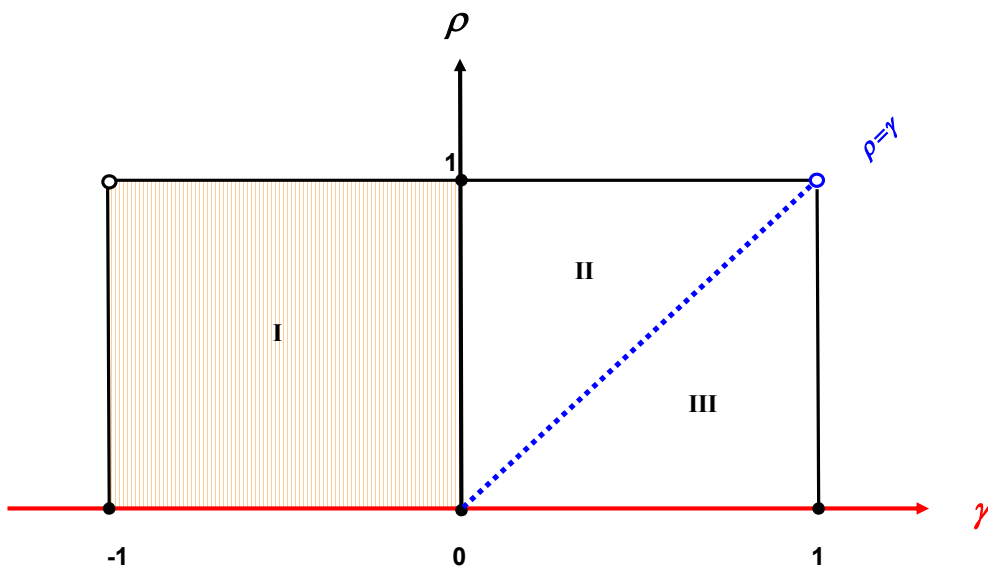
The partner of the public accounting firm supervises two managers and $m+k+n$ other employees (specialists, seniors and staff) for two client engagements. Engagement 1 has m employees working exclusively on client 1, Engagement 2 has n employees working exclusively on client 2, and both the engagements share k employees working on both the clients. Hence, we use ratio $k/(m+k+n)$ to represent the degree of uniformity of audit judgment processes applied to both the audit engagements.

Figure 2: The benefit of sharing knowledge varies with ρ and γ



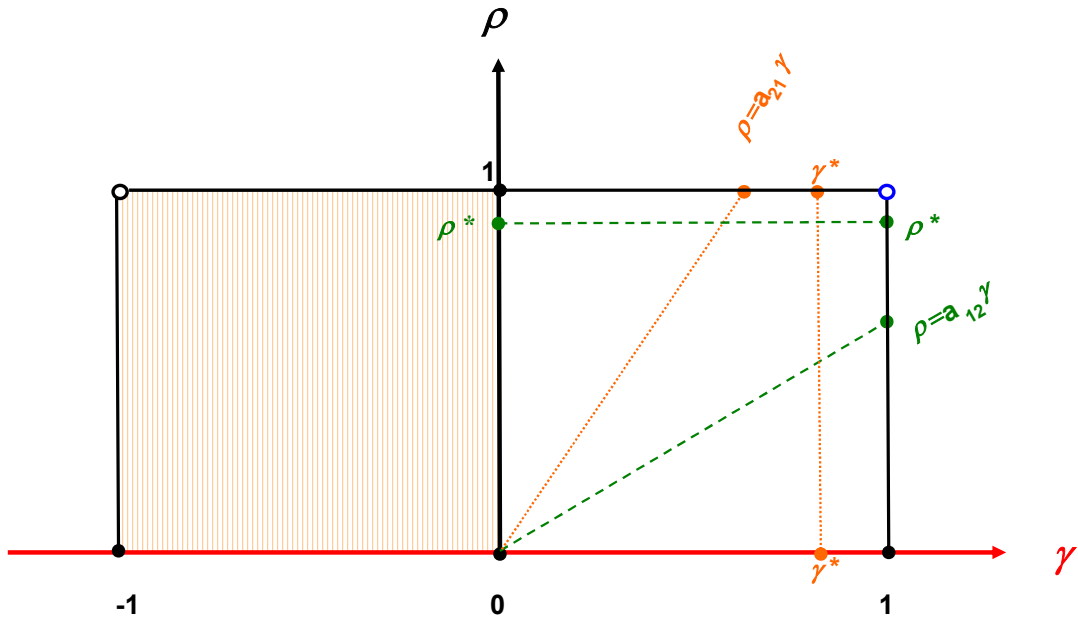
Sharing knowledge generally improves audit quality. However, there are four special cases in which such improvement due to sharing knowledge will *not* occur. These special cases are identified by the two dotted rays and one dashed ray, which all start from the origin (see Proposition 1 for details).

Figure 3: The audit quality varies with ρ and γ



The audit quality is increasing in ρ and decreasing in γ in Regions I and II, is decreasing in ρ and increasing in γ in Region III, and is independent of ρ and γ along with the 45-degree line between Regions II and III.

Figure 4: The audit quality and the knowledge-sharing benefit vary with parameters ρ and γ



The audit quality and the knowledge-sharing benefit may be either increasing or decreasing in parameters ρ and γ , depending on how ρ and γ relate to each other (see Proposition 4 for details).

Table 1: The overall impact of audit staff overlap on audit quality

Conditions	$\frac{dq_i}{dk}$	=	$\frac{\partial q_i}{\partial \rho}$	$\frac{\partial \rho}{\partial k}$	+	$\frac{\partial q_i}{\partial h_i}$	$\frac{\partial h_i}{\partial k}$	+	$\frac{\partial q_i}{\partial h_j}$	$\frac{\partial h_j}{\partial k}$	
If $c = 0$	Positive			Zero		Positive	Positive		Positive	Positive	
If $\gamma = 0$ and $\rho > 0$	$\frac{dq_1}{dk}$	Positive	=	Positive	Positive	Positive	Positive, if $m > c$	+	Negative	Negative, if $n < c$	
	$\frac{dq_2}{dk}$		=				Positive, if $n > c$	+		Negative, if $m < c$	
If $\gamma > 0$ and $\rho > 0$	$\frac{dq_1}{dk}$	Positive	=	Positive, if $\rho > \rho^\circ$	Positive	Positive	Positive, if $m > c$	+	Negative, if $\rho > \rho^\circ$	Negative, if $n < c$	
		Negative	=	Negative, if $\rho < \rho^\circ$			Negative, if $m < c$	+	Positive, if $\rho < \rho^\circ$		
	$\frac{dq_2}{dk}$	Positive	=	Positive, if $\rho > \rho^\bullet$			Positive, if $\gamma \leq \gamma^*$	Positive, if $n > c$	+	Negative, if $\rho > \rho^\bullet$	Negative, if $m < c$
		Negative	=	Negative, if $\rho < \rho^\bullet$ & $\gamma \leq \gamma^*$				Negative, if $n < c$	+	Positive, if $\rho < \rho^\bullet$	