Marketing Applications for Variance Analysis: The Need for a New Variance Model

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ABSTRACT

Variance analysis is based upon arguments and assumptions made over fifty years ago. As variance analysis moves into new fields, such as marketing, and new applications, such as non-financial performance measures, the validity of these traditional assumptions needs to be reviewed. This paper argues these assumptions do not hold in marketing applications, and thus, a new variance model is needed to avoid the calculational errors resulting from our traditional model. This new model, based upon the geometry of variance analysis, correctly calculates both the primary and residual (joint) variances in all four economic scenarios. The model is illustrated in a traditional production cost example, and in two common marketing applications.

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INTRODUCTION

Over seventy-five years ago Henry Maynard (1927, 562) wrote of variance analysis, "Its essential value lies in the fact that it is a control system." About fifty years ago, accounting researchers debated the algebra of variance analysis (Amerman 1953; Banerjee 1953; Vance 1950; Watson 1960; Weber 1963). While each researcher identified a flaw in the algebra, unique to a particular economic scenario, and proposed a solution, none investigated all four economic scenarios that might exist. Thus, no general algebraic model resulted that can be universally applied.

Instead, these researchers developed assumptions allowing their models to work in the limited scenario each identified. The two primary assumptions are, (1) small errors due to the allocation of small residual variances should be of little concern, and (2) the conventional two-variance solution provides the correct solution in most practical (manufacturing) cases (Amerman 1953). This led Vance (1950, 625) to conclude, "The formulas which are given in standard textbooks for variance analysis represent conventional, rather than logical, analysis." His observation remains valid with respect to the modern treatment of variance analysis.

The basic premise of variance analysis is that larger variances are symptoms of larger control problems. Outside of the traditional production applications, though, are control situations plagued with large joint variances and large potential errors. This implies that the first assumption must be reviewed. Further, this paper will demonstrate that conventional two-variate flexible budget analysis inflates variances in three out of four possible economic scenarios, which forces a review of the second assumption. This paper also shows the traditional three-variance solution, upon which the conventional variance solution is based, is equally flawed. The traditional debate about the efficacy of the three-variance solution over the practical simplicity of the two-variance solution is made moot when it is realized both are inaccurate.

This paper has three goals. The first is to review the traditional algebra and geometry of variance analysis behind the universal adoption of the flexible-budget model. The second goal is to present a modified decomposition model resolving issues associated with erroneous decision-making due to inflated variances in the conventional analysis. Third, this new model will be used in two common marketing applications.

BACKGROUND AND LITERATURE REVIEW

The Geometry of Variance Analysis

The logic of variance analysis is to explore the impact from changes in one variable while holding the other variable constant. The geometry of variance analysis is illustrated in Figure 1.

Insert Figure 1 here

To understand the primary variances, it is useful to imagine a rectangular clay tablet A, with the length of one side representing the actual price and the length of the other side representing the actual quantity. Tablet A's area represents the total actual cost, and includes Areas 1 and 3 in Figure 1. A second clay tablet, B, represents the total budgeted cost (Areas 1 and 2).

The two tablets overlap (Area 1). Area 2 is the primary quantity variance, and Area 3 is the primary price variance. In Figure 1, there is no residual or joint variance. The difference between the actual cost and the budgeted cost is equal to the sum of the two primary variances.

In Figure 2, actual cost is greater than budgeted cost. Area 1 represents the budgeted cost, $C_b = P_bQ_b$, and can be imagined as a tablet resting upon another tablet representing the actual cost.

The sum of areas 1, 2, 3 and 4 represent the actual cost, $C_a = P_aQ_a$. The difference between the two total costs, $C_a - C_b$, is the sum of areas 2, 3, and 4.

Insert Figure 2 here

Area 2 again represents the primary quantity variance; the change in cost caused by the change in quantity, while holding price constant at P_b = \$1. Area 3 again represents the primary price variance; the change in cost caused by a change in the purchase price, when holding quantity constant at $Q_b = 3$.

Area 2 is the same primary quantity variance with the same magnitude in both Figure 1 and Figure 2. Area 3 is the same price variance with the same magnitude in both Figure 1 and Figure 2. There is no difference in the absolute values or the primary variances. The primary variances provide the same magnitude of symptoms in Figure 1 and Figure 2. The only difference between the variances in Figure 1 and Figure 2 is the joint or residual variance.

There is no joint variance in Figure 1. In Figure 2, Area 4 is needed to calculate the difference between actual and budgeted cost. It represents the joint variance, and reflects the impact on cost of simultaneous or joint changes in both price and quantity.

From a managerial point-of-view, the relative sizes of the primary variances are the diagnostic focus of variance analysis because they identify the impact of one change at a time. The joint or residual variance is of less interest because it cannot be attributed to the change in a single variable.

The geometry in Figures 1 and 2 provides the basic logic and definitions used in the theory of variance analysis. An important feature of this geometry is that the size of each area remains constant regardless of a change in labels. That is to say, if the budgeted priced, P_b, is re-labeled to be the actual price, P_a, and vice-versa, the size of primary price variance remains the same.

We shall exploit this feature later in the paper.

It is obvious from Figure 1 that situations exist in which a joint variance should not be calculated (as in the three-variance model), or included in one of the primary variances (as in the two-variance model). Although in some situations (Figure 2) a joint variance must be calculated when explaining the difference between budgeted and actual cost. We have invoked this Pythagorean image of clay tablets to dispel the misconception that all variance analyses must have a joint or residual variance.

Deriving the Decomposition Equations

Textbooks fail to demonstrate the derivative of the basic two-variate decomposition equation and for completeness we review these basics following Amerman (1953),Watson (1960) and Kloock and Schiller (1997). The decomposition process is initiated by defining simple identities dealing with actual price, P_a, and actual quantity, Q_a, such that:

$P_a = P_a$ and,

$Q_a = Q_a$

The two identities are expanded by adding a constant representing budgeted price, P_b, and a constant representing budgeted quantity, Q_b, to produce:

$P_a + P_b = P_a + P_b$

$Q_a + Q_b = Q_a + Q_b$

These equations are re-ordered and simplified as:

$P_a = P_b + (P_a - P_b)$ or $P_b + \Delta P$

$Q_a = Q_b + (Q_a - Q_b)$ or $Q_b + \Delta Q$

Amerman (1953) substituted these equations into the actual cost equation to gain the general form of the basic decomposition equation expressed in terms of actual and budgeted values:

$$C_a = P_a Q_a$$

$$C_a = (P_b + (P_a - P_b))\ (Q_b + (Q_a - Q_b)) \qquad \#1$$

or: $$C_a = (P_b + \Delta P)\ (Q_b + \Delta Q)$$

Creating the General Three-Variance Solution

Amerman (1953) expanded equation #1 by multiplying the terms together and then reorganizing them to have the variance in total cost, $C_a - C_b$, represented by the two primary variances and the joint variance, such that:

$$C_a = P_b Q_b + P_b\ (Q_a - Q_b) + Q_b\ (P_a - P_b) + (Q_a - Q_b)(P_a - P_b)$$

$$C_a - C_b = P_b\ (Q_a - Q_b) + Q_b\ (P_a - P_b) + (Q_a - Q_b)(P_a - P_b) \qquad \#2$$

or: $$C_a - C_b = P_b\ \Delta Q + Q_b\ \Delta P + \Delta Q \Delta P$$

where:

C = total cost,

Q = total quantity of material purchased,

$P = C/Q$ = cost per unit of material purchase,

$Q_b\ (P_a - P_b)$ = a primary price variance,

$P_b\ (Q_a - Q_b)$ = a primary quantity or volume variance,

$(Q_a - Q_b)(P_a - P_b)$ = joint or residual variance

Subscripts: a = actual, b = budgeted.

The values of the three variances in equation #2 reflect the same values presented for each of the three areas illustrated in Figure 2.

Creating the Two-Variance Model

Amerman's (1953) method for deriving the decomposition equations is cumbersome, but it has the advantage of presenting it in a general form, which consists of two primary variances and a joint variance. Amerman also illustrated two possible versions for two-variate decompositions. He denoted the two procedures as Method 1 and Method 2 and distinguished them in terms of their joint variance allocations.[1] Vance (1950) and Watson (1960) argued the three-variance presentation was the general solution and that the conventional two-variance solution is a special case requiring extra assumptions.

Combining the primary price variance with the joint variance in equation #2:

$$C_a - C_b = P_b (Q_a - Q_b) + Q_b (P_a - P_b) + (Q_a - Q_b)(P_a - P_b) \quad \#2$$

$$C_a - C_b = P_b (Q_a - Q_b) + (P_a - P_b) (Q_b + (Q_a - Q_b))$$

$$C_a - C_b = P_b (Q_a - Q_b) + Q_a (P_a - P_b) \quad \#3$$

or: $C_a - C_b = P_b \Delta Q + Q_a \Delta P$

The transformation from equation #2 to #3 is accomplished by combining the joint variance with the primary price variance, $Q_b (P_a\text{-}P_b)$ and thus, creating a new definition of the price variance,

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[1] Combining the primary quantity variance with the joint variance in equation #2 produces Method 2:

$$C_a - C_b = P_b (Q_a - Q_b) + Q_b (P_a - P_b) + (Q_a - Q_b)(P_a - P_b) \quad \#2$$
$$C_a - C_b = (Q_a - Q_b) (P_b + (P_a - P_b))+ Q_b (P_a - P_b)$$
$$C_a - C_b = P_a (Q_a - Q_b) + Q_b (P_a - P_b)$$
or: $C_a - C_b = P_a \Delta Q + Q_b \Delta P$

where:

$P_a (Q_a - Q_b)$ = the quantity or volume variance

$Q_b (P_a - P_b)$ = the budget price variance

It has been suggested by Weber (1963) that Camman used this model as the foundation for his costing system in 1932, but it failed to gained popularity among practitioners. Amerman (1953) used permutation theory to prove that any two-variate model of variance has only two alternative methods of decomposition and method 1 represents the conventional practice based on the flexible-budget. Thus, method 2 is not considered further in this paper.

$Q_a (P_a - P_b)$. The shift from a definition based on budgeted quantity, Q_b, to actual quantity, Q_a, is a crucial, albeit subtle, change. The simplification from a three to two-variance solution produces the conventional two-variate flexible-budget analysis based on actual quantities that is universally used in Management and Cost Accounting texts. That is to say, equation #3 is the decomposition equation universally used in flexible budgeting but never discussed in modern textbooks.

The two-variance model and the three-variance model lead to two different solutions:

The Three-variance Solution:

Price Variance:	$Q_b(P_a - P_b) = 3(\$2 - \$1) = \underline{\underline{\$3}}$
Quantity Variance:	$P_b(Q_a - Q_b) = \$1(4 - 3) = \underline{\underline{\$1}}$
Residual Variance:	$(Q_a - Q_b)(P_a - P_b) = (4 - 3)(\$2 - \$1) = \underline{\underline{\$1}}$

The Two-variance Solution:

Price Variance:	$Q_a(P_a - P_b) = 4(\$2 - \$1) =$ **$\underline{\underline{\$4*}}$**
Quantity Variance:	$P_b(Q_a - Q_b) = \$1(4 - 3) = \underline{\underline{\$1}}$

*Note how the two-variate model inflates the price variance with the joint variance. Although the practice of using the two-variate model is well established, the manner in which the joint variance is allocated to one of the primary variances has been controversial for many years. “It leads to erroneous adjustment of variance (Vance 1950, 625).”

Vance (1950, 627) suggested that “statements (created by the two-variate model) are in error because they fail to recognize that the joint variance exists” and “that the joint product is conventionally thrown in with the price variance is apparently due to the traditional acceptance of the original error."

Watson (1960) also believed the two-variance method was wrong and advocated the three-variance solution, arguing this bias leads to a potential distortion in corrective control decisions. Kloock and Schiller (1997) criticized the two-variance model because its lack of symmetry leads to difficulties in cost-reduction planning and in the assignment of cost overrides to managers and departments.

Reasons for the General Acceptance of the Two-Variance Model

Apparently, two related causes led to the general acceptance of the traditional two-variance model taught in our current Management and Cost Accounting texts. One was the first industrial revolution and the Scientific Management strategy developed to organize work in the new capital-intensive factories. Another was the emphasis on external financial reporting in this country.

To support the development of large, capital-intensive factories during the first industrial revolution, significant investment capital was needed. Thus, top managements desired information about investment efficiency. Because these investments were directed to conversion processes (converting materials and labor into manufactured products), cost accounting systems evolved to provide detailed information about the manufacturing costs of products.

Due in part to the labor environment (i.e., a not-highly-educated labor force willing to work for low wages, but one highly motivated to work), Scientific Management became the dominant strategy for organizing work. Value chain activities were broken-down into quickly and easily taught tasks (e.g., shoving coal), and departments created for controlling similar activities (e.g., welding or painting departments).

Thus, a fundamental axiom of Scientific Management became the "separation-of-duties" principle. Each department was developed as a separate "functional silo with a goal to minimize production costs (especially the large capital investments and resulting fixed costs of the factory and equipment). Another principle was to keep workers busy all the time (maximizing economies-of-scale). The separation-of-duties principle also led to a greater separation of management and labor. Often, a company's headquarters (top management) was not located near its factories. To provide formal information about the efficiency of its conversion processes, cost accounting systems measuring the manufacturing value chain activities were developed.

Through techniques such as time-and-motion studies, industrial engineers were able to develop the "one best way" to perform each task. Performance standards (standard times and quantities) and measured variances from them logically followed. Because each department was a functional silo, operating independently from other departments, measuring efficiency through departmental cost variance reports dominated the cost accounting system (e.g., Harrison's 1918 set of equations for analyzing cost variances). Using cost variances to evaluate performance and motivate efficiency gains, the cost accounting system became the company's management accounting system (Johnson and Kaplan 1987). Thus, the algebraic approach to variance analysis became the accepted pedagogy and practice, and its underlying geometric reality disappeared from our texts.

Through the interaction with a related cause (i.e., this country's emphasis on external financial reporting), the algebraic approach became entrenched. To raise the financial capital needed during the first industrial revolution, investors purchased stock in the manufacturing companies. Especially since the late 1920's (and the American stock market crash), the investing public demanded accountability for management's stewardship role. Accountability was evidenced in publicly available financial reports, whose reliability was assured through audits by certified public accountants. To assure the financial statements were accurate, auditors required report articulation through a transaction-based financial accounting system following generally accepted accounting principles.

This resulted in the need for the product's "cost" to be objectively verified through a transaction-based journal entry recording system. Thus, the cost accounting system became a system for attaching costs to products, departments and managers with a focus on reducing cost. Because financial statements report costs by resource (materials, labor, and overhead, versus for example, reporting costs by activities), algebraic equations were needed to calculate and journalize resource cost variances. Through journalized cost-attaching, financial accountants could provide a fully absorbed product cost within a simple-to-install and operate system that also was simple to understand (Johnson and Kaplan 1987, 187). Most importantly, though, such a system satisfied external reporting requirements. Using the standard cost systems developed with Scientific Management, the need for a simple two-variance solution and journalized cost variances was reinforced, and the algebra of the flexible-budget became the accepted model.

Management and cost accounting systems could have developed independently from the financial accounting systems, in order to provide more accurate models of variance analysis. The three-variance algebraic model was recognized as the general equation. However, the joint variance was difficult to calculate in the paper-and-pencil environments of the early and mid-twentieth century. It also was not readily interpretable and assignable to a single department or manager, and defied systematic journal entries. Thus, practical arguments were used to reject the three-variance model, in favor of the workable two-variance approach in the flexible-budget model (Vance 1950). Probably the most pervasive practical argument for the two-variance approach came from Amerman (1953), who stated that if standards are current and variances small, joint variances should be of little concern. Thus, arbitrarily attaching the joint variance to the primary price variance will not significantly distort the reported results.

Educators quickly adopted Amerman's argument supporting practice, and Cost Accounting courses were taught as a series of journal entries to support inventory valuation methods, with little attention paid to their decision-making and motivational relevance (Johnson and Kaplan 1987). Thus, education became practitioner-driven, and professional schools of accountancy developed. This pedagogical strategy also supported the primary role for accounting education in this country (i.e., to produce CPA's). As an example, even in 2004, producing CPA's is the publicly stated, primary goal for UCLA's accounting program. Thus, the dominance of financial accounting systems and the Scientific Management strategy in this country resulted in, even to this day, a pervasive belief among financial accounting academics that Management Accounting and Cost Accounting are synonymous, and cost accounting systems exist primarily to support external financial reporting.

By the 1950s, the algebraically-driven pedagogy and practice were entrenched. But, some researchers turned their attention to relevance and developing separate management accounting systems to support managerial decision-making needs. However, management accounting research often focused on the application of economic, operations research, and mathematical models to accounting problems. According to Johnson and Kaplan (1987), operations research became the successor to Scientific Management in driving cost accounting systems development. This research, though, was communicated only between academics, with virtually no attention paid to practitioner and pedagogical needs. "Thus, the models were not developed for or tested on actual enterprises (Johnson and Kaplan 1987, 172)." Exasperating this "disconnect," practitioners were not writing about real needs, innovations, or problems in publications such as *Management Accounting*.

This partly explains why the 1950s – 1960s research on the algebraic approaches did not influence pedagogy or practice, and problems with their application were quickly forgotten. Another reason also contributes to this outcome. Around the 1950s, complex, multi-firm and multi-product industrial giants began to develop. To measure these organizations' profitability, research attention turned to more global measurement systems (e.g., ROI), and the management accounting system goals changed from process efficiency to overall firm profitability (Johnson and Kaplan 1987).

Other scholars have sought to design a compromise procedure maintaining the simplicity of the conventional two-variance model, but with a "valid" method of allocating the joint variance. Weber (1963) presented a summary of the various allocation methods, but all the alternatives had some disadvantage. The most damning of which is the argument that all allocations are arbitrary, and without a normative basis for allocating the joint variance, no two-variance model can be theoretically justified

Its only justification is in practice. It is simple, and the joint variance has no obvious interpretation for management. The joint variance by definition is caused by changes in both performance variables and does not guide the investigation towards one or the other of the primary variables.

While some researchers want to continue the debate on allocating the joint variance, it is a moot point. Whether the joint variance is allocated or not, and whether the two-variance or three-variance model is used, in three of the four possible economic scenarios, either model yields erroneous results. The root of the problem is much deeper than the problem of allocating a joint variance to one of the primary variances. The true root of the problem lies in how the original decomposition formula (equation #2) treats the primary variances differently in each of the four economic scenarios. That is to say, when equation #2, which generates a joint variance, is applied to a situation that does not have a joint variance, such as the one illustrated in Figure 1, errors in the sizes of variances result. Equation #2 always generates a joint variance whether or not it actually exists.

However, as we are now finally recognizing, traditional variance reports actually decrease productivity as managers and workers continue to waste time trying to explain ex-post variances having little to do with their economic reality (Johnson and Kaplan 1987). As management accounting systems design expands into new frontiers, such as marketing management, the problems created by the algebraic approach, which were argued away in production applications, are now resurfacing.

A NEW MODEL FOR VARIANCE ANALYSIS

The Four Economic Scenarios

Four scenarios can result from the combination of actual price and quantity with budgeted price and quantity, as shown in Figure 3.

Insert Figure 3 here

While Vance (1950) was the first to demonstrate how variance analysis produced different types of errors under different cases, Amerman (1953) and Banerjee (1953) formally classified the four possible economic scenarios. However, all of these early authors failed to notice that the three-variance solution does not separate the joint or residual variance from the primary variances in all four situations. Amerman (1953, 263) applied the three-variance model to Case 1, but not to the other three cases. Banerjee (1953) criticized Amerman for the omission but failed to note the inflated variances in his own three-variance solution. Watson (1960) also failed to note the problem of inflated variances in the three-variance model because he provided different numerical examples for each of the four cases. In the comparisons that follow (Figures 4 – 7), the same numbers are used in each of the four cases so the sizes of the primary variances remain constant. Only the labels of "actual" and "budgeted" are changed from case-to-case. That is to say, the absolute size of the two primary variances represented by area 2 and area 3 remain $3 and $1 respectively. Figures 4 – 7 are presented after the development of the new model so its results can be compared with the traditional three- and two-variance models.

Errors When Using the Two- and Three-Variance Models

Table 1 summarizes both models' errors.

Insert Table 1 here

The three-variance model inflates at least one of the primary variances in three of the four cases. Equation #2 always generates a joint variance and in cases 2 and 3 this is wrong because no joint variance exists (area 4).

In all four cases, the algebra (equation #2), ensures that the sum of the three variances equals the total variance, but a correct summation is not sufficient for an accurate solution. To be considered a correct solution, the price variance and the quantity variance calculated with the decomposition equation must equal the absolute values found in the geometry of the situation. For example, the algebra in the three-variance solution found in Case 1 provides the correct measurements for the primary variances. However, the price variance is inflated in Case 2, the quantity variance is inflated in Case 3, and both of the primary variances are inflated in Case 4. An inspection of the geometry demonstrates the source of the inflated variances is the inclusion of the joint variance. The algebra (equation #2) that produces the traditional three-variance solution inflates a primary variance in three of the four possible cases of two-variate analysis.

Kloock and Schiller (1997, 318) note the conventional two-variance solution arbitrarily allocates the joint variance calculated in the three-variance solution to the price variance and then point-out, “There is no theoretical justification for doing so.” Given that the three-variance model produces joint variance calculation errors in three of the four economic scenarios, it is not surprising to realize the two-variance model also produces incorrect variances in three of the four cases. Banerjee (1953, 352) noted the inflation problem in the two-variance model when he wrote, “In all these (three) cases the method gives wrong results.”

The fact that the three-variance model inflates the primary variances disqualifies it as a general solution for general variance analysis. The two-variance model, widely adopted in practice, is a special solution derived from assumptions in the three-variance model. Thus, the conventional two-variance flexible-budget model is not a general solution, and it is further disqualified because it can compound the original inflation problem. The true roots of its inaccuracies are not in the joint variance allocations through equation #3. The true roots of biased measurements are in the original derivation of equation #2.

The “Minimum Potential Performance Budget” Solution

If variance analysis is to be widely adopted outside the world of production control and cost accounting, a new procedure for calculating unbiased variances is needed. In marketing environments the standards and forecasts used in budgets are not as tight as they are in production. The inaccurate standards imply large variances, and large variances imply large joint variances, and large joint variances imply large potential errors due to inflated variances.

The central problem is the lack of correspondence between the geometry and algebra of each case. It is the desire to apply a single algebra to all four cases that is the source of the error. The general solution is to have a different equation for each of the four cases.

The use of four different equations to generate accurate measures of the primary variances is too awkward for practical use, however. Fortunately, all four have important properties in common that allow for creation of a single equation. The most important of these properties is that all use the minimum potential value as the level to hold a variable constant when making independent changes in the other variable. The second property is that all the independent changes in a variable are calculated in the conventional direction of budgeted value subtracted from actual value. The third property is that all four equations can be defined as having a residual variance as long as the residual variance is always equal to zero in cases 2 and 3.

The decomposition equation that correctly deals with all four cases is a three-variance solution modeled on the common properties of the four distinct equations, such that:

$$C_a - C_b = P_x (Q_a - Q_b) + Q_x (P_a - P_b) + r \qquad \#4$$

where:

C = total cost,

Q = total quantity of material purchased,

P = cost per unit of material purchased,

$Q_x (P_a - P_b) = (P_aQ_x) - (P_bQ_x)$ = a primary price variance,

$P_x (Q_a - Q_b) = (P_xQ_a) - (P_xQ_b)$ = a primary quantity or volume variance, r = joint or residual variance unexplained by the independent changes

Subscripts a = actual, b = budgeted, x = minimum of a, b.

Equation #4 correctly calculates the sizes of the primary variances. The residual variance that is not explained by the independent changes in the primary variances, is calculated following Bashan, et. al., (1973, 792):

$$r = C_a - C_b - P_x (Q_a - Q_b) - Q_x (P_a - P_b) \qquad \#5$$

In cases 2 and 3 the definition of the primary variances ensures the residual variance as calculated by equation #5 will equal zero. In cases 1 and 4, the residual variance equals the joint variance, and the algebra of equations #4 and #5 is consistent with the geometry of each economic scenario. This can be seen by comparing the traditional variance models with the proposed model in Figures 4 – 7.

Insert Figures 4 – 7 here.

From the geometry in Figures 4 – 7, the residual variance, Area 4, can never included in the price variance when using equation #4. When the minimum of the two quantity values is used as the constant for computing the price variance, then the price variance formula, $Q_x (P_a\text{-}P_b)$, is Area 3.

The same argument can be made for calculating the quantity variance. The quantity variance never includes the joint variance, Area 4, if the minimum of the two price values is used as the constant in calculating the quantity variance corresponding to the size of Area 2.

EVALUATING MEDIA MANAGERS USING THE MPPB MODEL

The purpose of this example is to illustrate the new model in the control of advertising plans. In the field of media planning, the term variance is appropriately understood as the magnitude of impact on an overall advertising goal due to a change in advertising activities.

The new Minimum Potential Performance Budget (MPPB) model is applicable to two-variate planning models used in advertising. That is to say, if overall advertising performance, Z, is expressed as the product of two advertising activities, X and Y, then the impacts on the overall goal due to the deviations in each activity (X or Y) can be isolated, measured, and compared. In more formal terms:

$$Z_a - Z_b = X_a Y_a - X_b Y_b = X_m (Y_a - Y_b) + Y_m (X_a - X_b) + r \qquad \#6$$

where:

$Z_a - Z_b$ = the difference between the performance goal and the actual results,

$X_m (Y_a - Y_b)$ = Y variance or the impact due to the deviation in activity Y, #7

$Y_m (X_a - X_b)$ = X variance or the impact due to the deviation in activity X, #8

$r = Z_a - Z_b - X_m (Y_a - Y_b) - Y_m (X_a - X_b)$ = joint variance or residual impact due to the simultaneous deviations in X and Y, #9

(subscripts: a = actual result, b = planned result, m = the minimum of a or b)

- Equation #6 is the MPPB model in which the difference between the plan's goal and the actual result is expressed as the sum of the impacts due to the individual deviations in advertising activities.
- Equation #7 represents the impact on the change in overall performance due to changes in activity Y, while activity X is held constant (the primary Y variance).
- Equation #8 represents the impact on the change in overall performance due to the deviation in activity X while activity Y is held constant (the primary X variance).

- ♦ Equation #9 is the impact on the goal due to the simultaneous deviations in activities X and Y (the joint variance).

Media advertising campaigns often are evaluated on their gross ratings points. Gross ratings points are calculated by multiplying frequency by reach. To provide an example, an advertising plan calls for 240 gross rating points (GRP_b) by achieving a frequency of 4 exposures per household and reaching 60% of households. At the end of the campaign, the media planner results show advertising frequency has deviated from plan by 25% and the reach has deviated by 30%. The actual reach was 78% and the actual exposure frequency was 3, resulting in a total of 234 gross rating points (GRP_a). This performance is summarized in Table 2.

Insert Table 2 here.

Which of the two deviations from plan is having the greatest impact on the change in advertising performance? Observation leads us to believe it is reach, because its variance is 30% compared to the frequency variance of only 25%. However, this is incorrect. The deviation from the planned frequency is the correct answer because it has the largest impact on the overall GRP performance.

Using the MPPB model, the media planner can accurately identify which of the two deviations in his advertising plan is having the greatest impact on GRP, as seen in Table 3.

Insert Table 3 here.

In this situation, the report conveys to the media planner that the 25% decrease in frequency lowered the overall performance by 60 gross rating points. The 30% percent improvement in the number of households reached increased the overall performance by 54 gross rating points. The net effect of the two deviations was a decrease of 6 gross rating points. On the basis of the variance report, the media planner now knows he must put priority into finding the causes behind the decrease in the frequency of his exposures per household rather than the reasons behind the increase in households reached. If the media planner can increase the frequency while maintaining his current reach, his overall gross rating points will increase more than with the converse strategy.

Because variance analysis is an alien concept to most media planners, they rely on experience and judgment in determining what to focus on to improve performance. Media planners have not had a model of variance analysis that is sufficiently accurate to test their judgments across the full range of possible media situations. The full range of scenarios specifically includes differences that can exceed budget or be short of budget. Each case is analogous to the four economic situations previously illustrated as Cases 1 – 4 in Figures 4 – 7. Table 4 illustrates each of the economic scenarios for this example. Table 5 provides a comparison between the variances calculated using the traditional three-variance model and the MPPB model.

Insert Tables 4 and 5 here.

EVALUATING SALES REVENUES USING THE MPPB MODEL

One of the most important decisions salespeople make on a day-to-day basis involves the abandonment of prospects and the initiation of new contacts. Salespeople have limited amounts of time and must choose to continue the selling process with a current prospective customer or drop the attempt and start a new lead with a new prospect. This is a very difficult task and many salespeople keep records of the number of prospects visited in a period and analyze their rate of sales-per-prospect contacted for clues about improvement.

There are many variations of this problem. For example, sales per day, calls per prospect, conversion rates, and batting averages are different measures used to explore a person's sales effectiveness. For the purpose of this paper we will concentrate on the rate of sales per prospect or potential account and the number of accounts or prospects visited. The conclusions reached about the methodology used in this paper can be applied to different variations of the sales evaluation problem.

The classic approach to the evaluation of salespeople involves the comparison of the individual's performance against a standard, quota or average performance. If the person's performance is deemed to be significantly above or below standard or average, an investigation into the causes of the specific deviation is called for. The focus of the conventional comparison is the deviations in the actual inputs and activities (e.g., the difference in the number of days worked, the number of prospect visited, the sales per call, the orders per lead, etc).

To illustrate, the sales performances of four salespeople are presented in Table 6.

Insert Table 6 here.

The classic report provides measures of the deviation from quota in terms of actual magnitudes and percentage changes. The overall measure of output is the sales revenue produced by each salesperson. The input measures are the total number of different prospects visited over the period and the rate of sales earned per prospect.

In the minds of many people, the number of prospects being visited is a measure of the salesperson's effort, and the average sales per prospect is a measure of the salesperson's effectiveness at converting leads into revenues. For the purposes of demonstration, the four performances in Table 6 have been contrived to have every salesperson with the same absolute deviations from the planned quota.

The manager can see that Jack and Bill have both worked harder than Sam and Fred by calling on more prospects. She also can see that Jack and Sam have worked more effectively than Bill and Fred by producing more revenue per prospect. However, the manager finds nothing in the conventional performance evaluation to help her establish which of the deviations from quota is having the largest impact on sales revenue. That is to say, there is nothing to indicate how many dollars of revenue can be attributed to Jack's increase in sales effort or how many dollars of sales revenue were lost because Fred did not reach quota on the number of prospects visited.

The absence of any information on the dollar impact on overall revenue due to deviations in sales activities may lead some to conclude, erroneously, that equal percentage changes on inputs imply equal impacts on outputs. In other words, a 20% change in number of visits can have very different impacts on overall sales performance. For example, Jack's 20% improvement over quota increases his overall sales revenue by $75,000, but Bill's 20% improvement over quota has a smaller ($56,250) impact on sales. Both Bill and Jack have the same size deviations in conventional report, but their deviations have different impacts on overall performance.

The difference between actual sales revenue and budgeted performance, $S_a – S_b$, is decomposed into the sum of the two impacts due to input deviations as shown in equation #10:

$$S_a - S_b = N_m(R_a - R_b) + R_m(N_a - N_b) + r \qquad \#10$$

where:

S = the total sales revenue

R = the rate of sales per prospect

N = the total number of prospects visited (leads processed)

$N_m(R_a - R_b)$ = the impact on sales due to deviation in sales per prospect

$R_m(N_a - N_b)$ = the impact on sales due to deviation in visited-prospects r = residual impact not explained by the two individual deviations

Subscripts a = actual, b = quota or budgeted, m = minimum of a, b

The value of residual impact is calculated as:

$$r = S_a - S_b - N_m(R_a - R_b) - R_m(N_a - N_b) \qquad \#11$$

These equations are the equivalent of equations #4 and #5 used to develop the MPPB model.

The values of the deviations are converted into the dollar impacts on overall sales performance using equations #10 and #11. The results are illustrated in Table 7.

Insert Table 7 here.

With the conventional report Bill and Jack appear to have missed their quota of sales per prospect by the same amount and may be misjudged to have performed equally. If their manager has the results presented in Table 7, she can see the dollar impacts on overall performance due to the deviations in individual activities. The impact due to Bill's drop in sales per prospect is having a more serious impact on overall performance than Fred's drop in sales per prospect. Both have the same 25% deviation but Bill's drop is having a $75,000 impact compared to Fred's impact of $56,250. The sum of the individual impacts equals the difference between the actual and budgeted revenues. However, the sales manager should focus on investigating the reason behind Bill's sales per prospect activities because they are having a larger impact than Jack's activities.

Some firms reward salespeople on the difference of their performances from average activity levels in terms of standard deviations. Jack and Bill have the same standard deviation when the activity is measured in terms of the input, number of prospects visited, but they have different standard deviations when measured from average impact on the output. If managers reward sales performances from the mean activity levels, then bonus or penalties ought to be awarded to individuals on the basis of the impacts on outputs.

Most performance evaluations take far more inputs into account than simply sales per prospect and number of leads serviced. The classic four-factor planning model found in most sales textbooks involves, days worked, D, calls per day, W, orders per call, B, and sales per order, Z (Spiro et al. 2003). The difference between actual and budgeted sales performance is written as:

$$S_a - S_b = Z_a B_a W_a D_a - Z_b B_b W_b D_b$$

This then is decomposed into the impact on sales due to individual deviations in the four sales factors, such that:

$$S_a – S_b = Z_m B_m W_m (D_a – D_b) + Z_m B_m D_m (W_a – W_b) + Z_m W_m D_m (B_a – B_b) + B_m W_m D_m (Z_a – Z_b) + r$$

where:

$S_a – S_b$ = difference between actual and planned sales performance

$Z_m B_m W_m (D_a – D_b)$ = impact due to deviation from planned level of working days

$Z_m B_m D_m (W_a – W_b)$ = impact due to deviation from planned rate of calls per day

$Z_m W_m D_m (B_a – B_b)$ = impact due to deviation from planned level of orders per call

$B_m W_m D_m (Z_a – Z_b)$ = impact due to deviation from planned order size

r = residual impact not explained by the impact of individual deviations

The contribution from this expansion is that the MPPB model can be easily expanded to account for more than two-variable situations.

DISCUSSION

The logic of variance analysis is to answer the question, if other variables stayed unchanged at the standard or budgeted level, then what impact on the total variance did the change in one particular variable have?

Managers use the relative sizes of the primary variances to compare activities within and across departments and operational boundaries. That is to say, a manager may wish to compare the price and quantity variances within a production process or compare price variances across product lines. The relative sizes of the primary variances are the symptoms of potential control problems. The assumption is the larger the primary variance the greater the potential control problem. The need for unbiased measures of the primary variances is the justification of the proposed solution. The proposed solution reports the impact of the joint variance separately from the impact of the primary variances.

The goal of the analysis is to calculate the primary variances in a way that ensures the exclusion of the joint or residual variances. The traditional two-variance solution based on the flexible budget provides unbiased measures in only one of the four possible cases. In 100 years of practice on the control of production costs, the errors created by the inappropriate application of the flexible-budget (equation #3) apparently have been small enough to be ignored. The size of the errors may be of little consequence when the standards are accurate and the variances are small. But, when variance analysis is applied to non-production arenas, the assumptions of good forecasts and small variances may not be valid, and a more accurate model, as proposed here, should be used.

When the geometry of variance analysis is considered, a new focal point is created. The new focal point is the minimum potential performance levels, P_xQ_x. The minimum potential performance value may or may not correspond to the conventional focus of actual results (P_aQ_a), static budgets (P_bQ_b), or flexible-budgets (P_bQ_a). The minimal potential performance is a new and consistent point of reference using the minimums of the actual or standard values.

The obvious issue with the proposed three-variance solution generated by equation #4 is the dissolution of the flexible-budget model as a universal constant. The minimum potential level of budget performance, P_xQ_x, replaces the traditional base of actual quantity times standard price (P_bQ_a). The interpretation of a minimum level of performance based on some potentially hypothetical combination of actual and budgeted values is disconcerting to accountants holding the flexible-budget as a concrete and universal constant. However, the flexible-budget is no less abstract than the minimum potential performance budget. It is the potential to have a different minimum at different times that is awkward for pedagogical purposes. However, any reification of budgets is due solely to common usage and practice.

From a pedagogical point of view the proposed solution makes it almost impossible to teach variance analysis using the traditional columnar format. The ability to assign a constant amount such as the flexible-budget value is lost for all four cases. The conventional columnar system based on flexible budgeting is simpler, but in new fields of application, accuracy is more important than simplicity. If the proposed solution is adopted in our textbooks, authors will be forced to rely more on algebraic and geometric presentations than columnar presentations.

As demonstrated in the last example application, this proposed solution has the advantage of simplifying the conventional multi-level or “onion-layer” approach to the decomposition of multiple variables. With the proposed solution, the complete set of variances can be analyzed and compared in a single level of presentation. In complex situations, the proposed system is considerably simpler than the conventional analysis and it lends itself to spreadsheet and database manipulations.

CONCLUSION

Accountants identified the mis-calculations resulting from the three and two-variance models many years ago, but dismissed the potential inaccuracies due to residual variances as offering "no reason for undue concern" (Amerman 1953, 266). In more recent years, these models have received criticism for inappropriately affixing blame for deviations to departments and activities (Bentz and Lusch 1980; Kloock and Schiller 1997). This paper proposes an alternative model based on the geometry of the four possible economic situations when comparing budgeted and actual results. The proposed MPPB model offers advantages, but also has disadvantages, as summarized below.

Advantages of the Minimum Potential Performance Budget Solution

1. It is easier to apply in non-production environments where outcomes are not measured in terms of prices and quantities.
2. The procedure produces unbiased measurements of the primary variances. That is to say, the measurement of the impact solely due to changes in the primary variables is isolated from the impact of the joint movement in several variables. The conventional two-variance solution adds the joint variance to the price variance.
3. It can be applied to a wider range of situations than the conventional flexible-budget procedure. For example, it can produce accurate measures in situations with inaccurate forecasts and large variances. Conventional analysis assumes that standards are current and variances are small.
4. The residual joint variances that are unexplained by the changes in the individual variances are reported separately.

5. The proposed procedure eliminates the difficulty of explaining the arbitrary assignment of joint variances to the responsible managers. The conventional system results in the arbitrary allocation of joint variances to one manager or another, and this is perceived as an unfair practice.

Disadvantages of the Proposed Procedure

1. The concept of a minimum potential performance budget, $P_{min}Q_{min}$, is more abstract than the concept of the flexible-budget, P_bQ_a. The minimum potential performance budget is not a rigid standard, but varies with forecasts and performances.
2. The residual variance adds to the complexity of the variance report, but provides no managerial insights for control. There is very little managerial interpretation that can be given to the size of the residual variance caused by the joint changes in variables.
3. From a pedagogical point of view the proposed solution makes it almost impossible to teach variance analysis using the traditional columnar format. The ability to assign a constant amount, such as the flexible-budget value, is lost for all four cases. The conventional columnar system based on flexible budgeting is simpler, but in new fields of application, accuracy is more important than simplicity. If the proposed three-variance solution is adopted for textbooks, authors will have to rely on algebraic and geometric presentations.

Issues for Future Research

1. While the concept of a minimum level of potential performance (Area 1) makes sense in marketing applications, does it make sense in production applications?

2. Are joint variances taught in Cost Accounting courses? If not, why not? Are they somehow not important in production applications, but important in marketing applications?
3. If the MPPB model is rejected in favor of either the traditional three or two-variance models, how can we refine them so that they produce accurate performance evaluation measures?
4. If the two-variance model is to be applied in situations using non-financial performance measures, which measure should be held constant at its actual value, and which measure should be held constant at its budgeted value?

This paper presents the inaccuracies in our current variance models. Both the two and three-variance models produce incorrect variances in three of the four possible economic situations that can result from comparing budgeted and actual performance. The correct analysis for each situation is geometrically demonstrated, and the MPPB equations derived from it.

Current cost accounting pedagogy and practice is to ignore the joint variance, resulting in its inclusion in the price variance. Theoretic problems identified a half century ago are now resurfacing as real practical problems in performance evaluations. When applied outside of cost accounting environments (e.g., in marketing applications), the need to calculate unbiased measures of the primary variances, and isolating the joint variance, are even more important. It is hoped the proposed MPPB model in this paper can be easily adapted to situations in which unbiased measures are needed, such as in non-financial marketing performance evaluations.

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Figure 1
Areas of Primary Variance

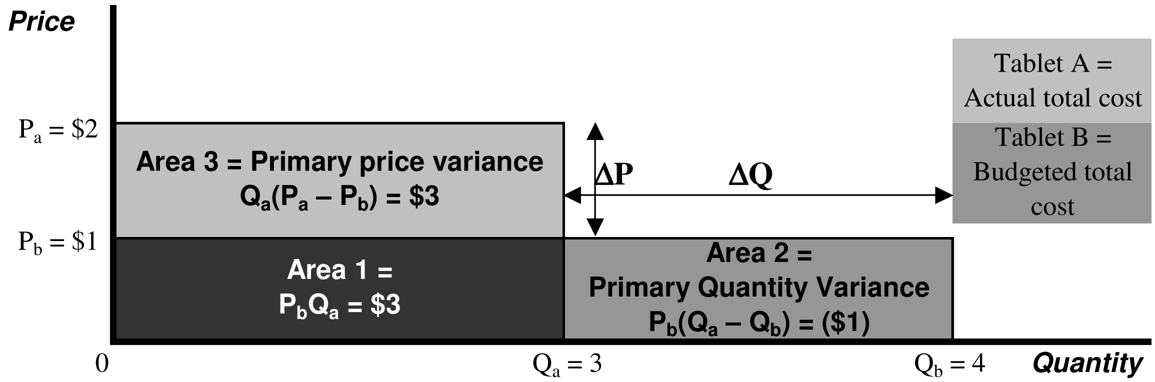


Figure 2
The Geometry of a Joint Variance

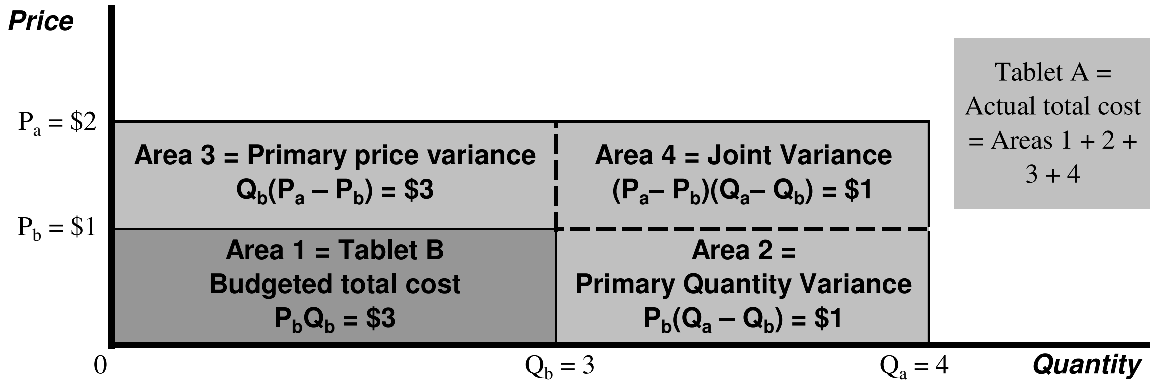


Figure 3
The Four Economic Scenarios

	$P_a > P_b$	$P_a < P_b$
$Q_a > Q_b$	**Case 1**	**Case 3**
$Q_a < Q_b$	**Case 2**	**Case 4**

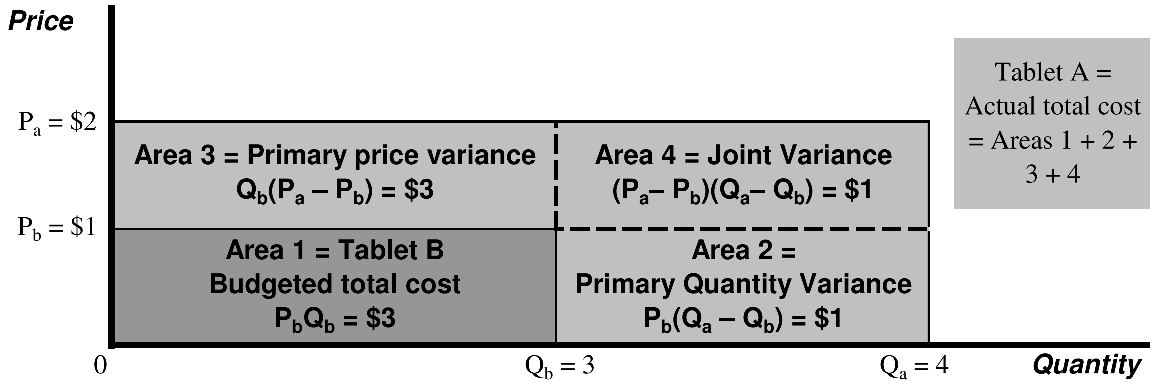
Table 1
Algebraic Variance Models' Errors

	$P_a > P_b$	$P_a < P_b$
$Q_a > Q_b$	**Case 1** 3-variance model: None 2-variance model: Price variance	**Case 3** 3-variance model: Quantity & Joint 2-variance model: Price & Quantity
$Q_a < Q_b$	**Case 2** 3-variance model: Price & Joint 2-variance model: None	**Case 4** 3-variance model: Price, Quantity, & Joint 2-variance model: Quantity

Figure 4

Case 1:
$P_a > P_b$ and $Q_a > Q_b$

The Geometric Solution:



The Three-variance Solution:

Price Variance: $Q_b(P_a - P_b) = 3(\$2 - \$1) = \underline{\underline{\$3}}$

Quantity Variance: $P_b(Q_a - Q_b) = \$1(4 - 3) = \underline{\underline{\$1}}$

Residual Variance: $(Q_a - Q_b)(P_a - P_b) = (4 - 3)(\$2 - \$1) = \underline{\underline{\$1}}$

The Two-variance Solution:

Price Variance: $Q_a(P_a - P_b) = 4(\$2 - \$1) = \underline{\underline{\$4^*}}$

Quantity Variance: $P_b(Q_a - Q_b) = \$1(4 - 3) = \underline{\underline{\$1}}$

The Minimum Potential Performance Budget Solution:

Price Variance: $Q_{min}(P_a - P_b) = 3(\$2 - \$1) = \underline{\underline{\$3}}$

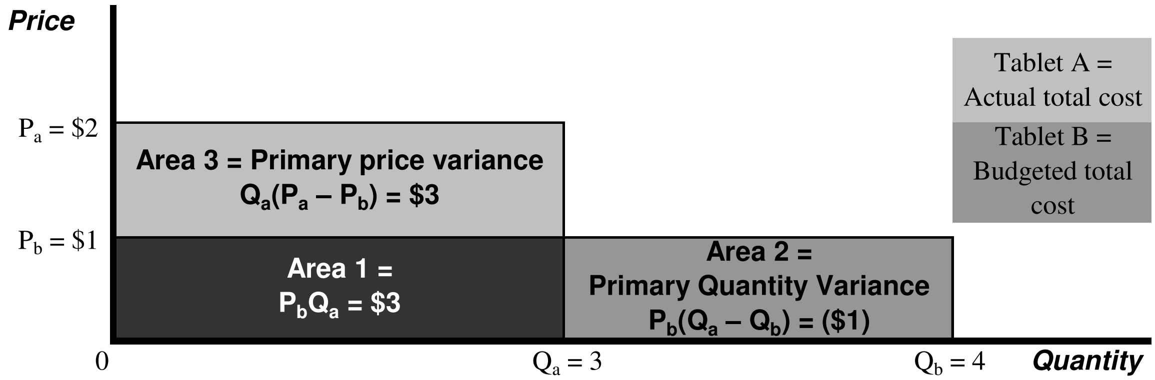
Quantity Variance: $P_{min}(Q_a - Q_b) = \$1(4 - 3) = \underline{\underline{\$1}}$

Residual Variance: $(C_a - C_b) - [Q_{min}(P_a - P_b)] - [P_{min}(Q_a - Q_b)] = \underline{\underline{\$1}}$

Figure 5

Case 2:
$P_a > P_b$ and $Q_b > Q_a$

The Geometric Solution:



The Three-variance Solution:

Price Variance: $Q_b(P_a - P_b) = 4(\$2 - \$1) =$ **$4***

Quantity Variance: $P_b(Q_a - Q_b) = \$1(3 - 4) =$ ($1)

Residual Variance: $(Q_a - Q_b)(P_a - P_b) = (3 - 4)(\$2 - \$1) =$ **($1)***

The Two-variance Solution:

Price Variance: $Q_a(P_a - P_b) = 3(\$2 - \$1) =$ $3

Quantity Variance: $P_b(Q_a - Q_b) = \$1(3 - 4) =$ ($1)

The Minimum Potential Performance Budget Solution:

Price Variance: $Q_{min}(P_a - P_b) = 3(\$2 - \$1) =$ $3

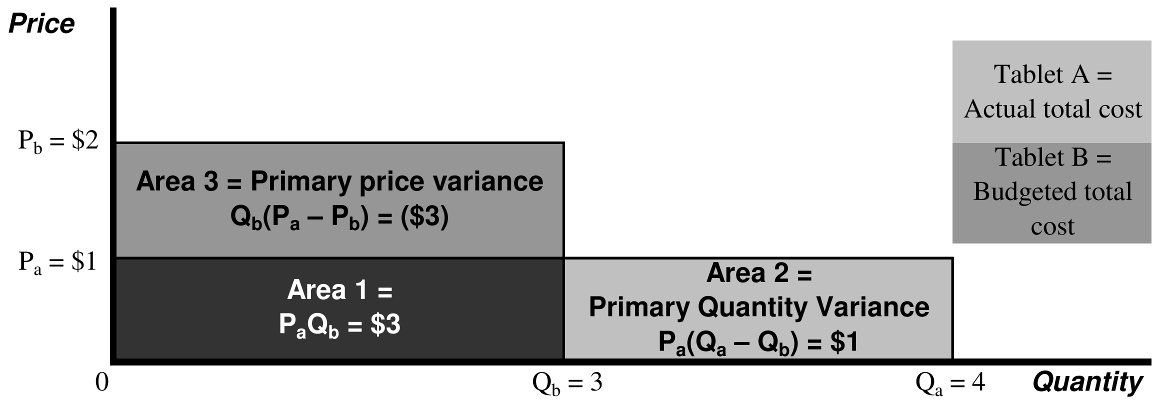
Quantity Variance: $P_{min}(Q_a - Q_b) = \$1(3 - 4) =$ ($1)

Residual Variance: $(C_a - C_b) - [Q_{min}(P_a - P_b)] - [P_{min}(Q_a - Q_b)] =$ $0

Figure 6

Case 3:
$P_b > P_a$ and $Q_a > Q_b$

The Geometric Solution:



The Three-variance Solution:

Price Variance: $Q_b(P_a - P_b) = 3(\$1 - \$2) = \underline{\underline{(\$3)}}$

Quantity Variance: $P_b(Q_a - Q_b) = \$2(4 - 3) = \underline{\underline{\$2^*}}$

Residual Variance: $(Q_a - Q_b)(P_a - P_b) = (4 - 3)(\$1 - \$2) = \underline{\underline{(\$1)^*}}$

The Two-variance Solution:

Price Variance: $Q_a(P_a - P_b) = 4(\$1 - \$2) = \underline{\underline{(\$4)^*}}$

Quantity Variance: $P_b(Q_a - Q_b) = \$2(4 - 3) = \underline{\underline{\$2^*}}$

The Minimum Potential Performance Budget Solution:

Price Variance: $Q_{min}(P_a - P_b) = 3(\$1 - \$2) = \underline{\underline{(\$3)}}$

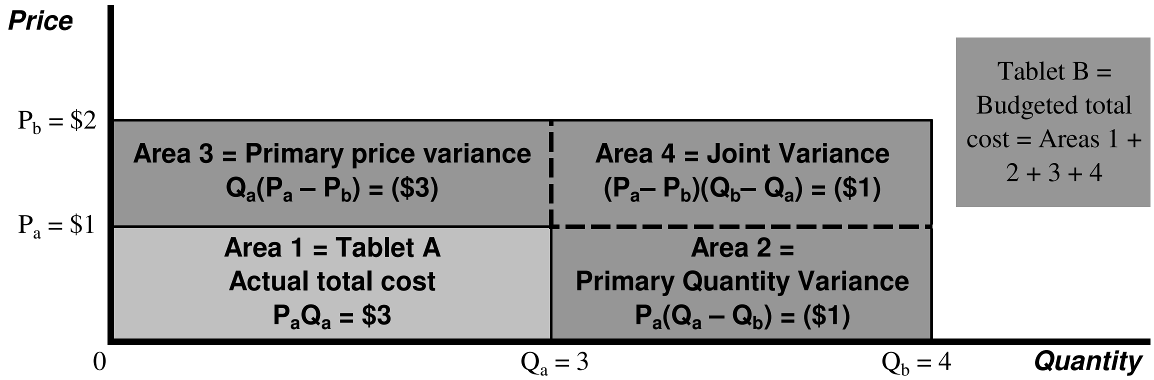
Quantity Variance: $P_{min}(Q_a - Q_b) = \$1(4 - 3) = \underline{\underline{\$1}}$

Residual Variance: $(C_a - C_b) - [Q_{min}(P_a - P_b)] - [P_{min}(Q_a - Q_b)] = \underline{\underline{\$0}}$

Figure 7

Case 4:
$P_b > P_a$ and $Q_b > Q_a$

The Geometric Solution:



The Three-variance Solution:

Price Variance: $Q_b(P_a - P_b) = 4(\$1 - \$2) =$ **($4)***

Quantity Variance: $P_b(Q_a - Q_b) = \$2(3 - 4) =$ **($2)***

Residual Variance: $(Q_a - Q_b)(P_a - P_b) = (3 - 4)(\$1 - \$2) =$ **$1***

The Two-variance Solution:

Price Variance: $Q_a(P_a - P_b) = 3(\$1 - \$2) =$ ($3)

Quantity Variance: $P_b(Q_a - Q_b) = \$2(3 - 4) =$ **($2)***

The Minimum Potential Performance Budget Solution:

Price Variance: $Q_{min}(P_a - P_b) = 3(\$1 - \$2) =$ ($3)

Quantity Variance: $P_{min}(Q_a - Q_b) = \$1(3 - 4) =$ ($1)

Residual Variance: $(C_a - C_b) - [Q_{min}(P_a - P_b)] - [P_{min}(Q_a - Q_b)] =$ ($1)

Table 2
Deviations in Advertising Plan

	Actual Results	**Budgeted Values**	**Deviations from Plan**
Gross Rating Points	$G_a = 234$	$G_b = 240$	– 6 GRP
Frequency = Number of Exposures per Household (F)	$F_a = F_m = 3$*	$F_b = 4$	– 1 exposure per household (25%)
Reach = Percentage of Households Reached (R)	$R_a = 78$	$R_b = R_m = 60$*	+18 (30%)
* Minimum level of each activity is labeled with subscript m			

Table 3
Variance Report — Reach and Frequency

Actual Gross Rating Points		234
Planned Gross Rating Points		240
Change in Gross Rating Points to be explained by deviations in reach and frequency activities		(6)
Impact on GRP due to the 30% increase in reach: $F_m (R_a\text{-}R_b) = 3(78–60)$	54	
Impact on GRP due to the 25% decrease in frequency: $R_m (F_a\text{-}F_b) = 60(3–4)$	(60)	
Residual impact on GRP due to the simultaneous changes in reach and frequency (r)	0	

Table 4
Advertising Example Data for Four Media Managers

	$F_a > F_b$			$F_a < F_b$		
	Case 1			**Case 3**		
		Actual	Budget		Actual	Budget
$R_a > R_b$	Frequency (F): (# of ads)	4	3	Frequency (F): (# of ads)	3	4
	Reach (R): (% of market)	78	60	Reach (R): (% of market)	78	60
	GRP:	312	180	GRP:	234	240
	GRP variance:	132 F		GRP variance:		6 U
	Case 2			**Case 4**		
		Actual	Budget		Actual	Budget
$R_a < R_b$	Frequency (F): (# of ads)	4	3	Frequency (F): (# of ads)	3	4
	Reach (R): (% of market)	60	78	Reach (R): (% of market)	60	78
	GRP:	240	234	GRP:	180	312
	GRP variance:	6 F		GRP variance:		132 U

Table 5
Comparing Variance Models for the Advertising Example

	$F_a > F_b$			$F_a < F_b$		
	Case 1			**Case 3**		
	Variances	3-Variance Model	MPPB Model	Variances	3-Variance Model	MPPB Model
$R_a > R_b$	Frequency:	60(4 – 3) = 60	60(4 – 3) = 60	Frequency:	60(3 – 4) = (60)	60(3 – 4) = (60)
	Reach:	3(78 – 60) = 54	3(78 – 60) = 54	Reach:	4(78 – 60) = 72	3(78 – 60) = 54
	Joint:	(78 – 60)(4 – 3) = 18	312 – 180 – 60 – 54 = 18	Joint:	(78 – 60)(3 – 4) = (18)	234 – 240 – (60) – 54 = 0
	Case 2			**Case 4**		
	Variances	3-Variance Model	MPPB Model	Variances	3-Variance Model	MPPB Model
$R_a < R_b$	Frequency:	78(4 – 3) = 78	60(4 – 3) = 60	Frequency:	78(3 – 4) = (78)	60(3 – 4) = (60)
	Reach:	3(60 – 78) = (54)	3(60 – 78) = (54)	Reach:	4(60 – 78) = (72)	3(60 – 78) = (54)
	Joint:	(60 – 78)(4 – 3) = (18)	240 – 234 – 60 – (54) = 0	Joint:	(60 – 78)(3 – 4) = 18	180 – 312 – (60) – (54) = 0

Table 6
The Performance Evaluations of Four Salespeople

	Jack	**Bill**	**Sam**	**Fred**	**Quota**
Sales, S Difference from Quota	$468,750 +$168,750	$281,250 –$18,750	$281,250 –$18,750	$168,750 –$131,250	$300,000
Prospects Visited, N Difference from Quota	75 +15 (20%)	75 +15 (20%)	45 –15 (–20%)	45 –15 (–20%)	60
Sales per Prospect R Difference from Quota	$6,250 +$1,250	$3,750 –$1,250	$6,250 +$1,250	$3,750 –$1,250	$5,000

Table 2
An Analysis of the Impact Caused By Deviations From Quota

	Jack	Bill	Sam	Fred
Difference between actual and planned level of sales to be explained, $S_a - S_b$	$168,750	–$18,750	–$18,750	–$131,250
Impact due to working harder i.e., due to deviation from planned rate of prospects visited $R_m(N_a - N_b)$	+15 visits 20% **$75,000**	+ 15 visits 20% **$56,250**	– 15 visits –20% **–$75,000**	– 15 visits –20% **–$56,250**
Impact due to working smarter i.e., due to deviation from the planned sales per prospect $N_m(R_a - R_b)$	+$1,250 25% **$75,000**	–$1,250 –25% **–$75,000**	+$1,250 25% **$56,250**	–$1,250 –25% **–$56,250**
Residual impact due to joint changes and not explained by individual deviations r	**$18,750**	**$0**	**$0**	**–$18,750**